

## Laplace transforms

$$L\{e^{\alpha t}\} = \frac{1}{s - \alpha}$$

$$L\{1\} = \frac{1}{s}$$

$$L\{t^n\} = \frac{n!}{s^{n+1}} \quad (n \text{ a nonnegative integer})$$

$$L\{\cos \beta t\} = \frac{s}{s^2 + \beta^2}$$

$$L\{\sin \beta t\} = \frac{\beta}{s^2 + \beta^2}$$

$$L\{e^{\alpha t} \cos \beta t\} = \frac{s - \alpha}{(s - \alpha)^2 + \beta^2}$$

$$L\{e^{\alpha t} \sin \beta t\} = \frac{\beta}{(s - \alpha)^2 + \beta^2}$$

$$L\{u_c(t)\} = \frac{e^{-cs}}{s} \quad \text{where } u_c \text{ is the step function which "switches on" at } t = c$$

$$L\{u_c(t)f(t - c)\} = e^{-cs}L\{f(t)\}$$

### Other information you may need:

Fundamental solutions to heat equation boundary value problems with homogeneous boundary conditions:

$$\sin\left(\frac{n\pi x}{L}\right) \exp\left(-\frac{\alpha^2 n^2 \pi^2}{L^2} t\right),$$

for positive integer values of  $n$ .

$$\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) + C$$

$$\int x \sin(ax) dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax) + C$$

In case the  $2 \times 2$  matrix  $A$  has complex eigenvalues  $r = \alpha + i\beta$  and  $\bar{r} = \alpha - i\beta$ , and  $\xi = u + iv$  is an eigenvector corresponding to eigenvalue  $r$ , the general solution to the system  $x' = Ax$  is

$$x(t) = e^{\alpha t} [c_1(u \cos \beta t - v \sin \beta t) + c_2(v \cos \beta t + u \sin \beta t)]$$