

Fall 2007 Exam 3

No calculator of any kind is permitted. Show all work and give clear explanations.

NAME:

Question	Points	Score	Pres Pt
1	25+1		
2	21+1		
3	25+1		
4	25+1		
Total	100		

1. (25+1 Points) Let f be the function which is identically equal to 1 over the interval $[0, 3]$. Find a Fourier sine series representation for f , good at least over $(0, 3)$.

2. (21+1 Points) Suppose we seek solutions $u = u(x, t)$ to the equation $u_x - t^2 u_{tt} = 0$.

(a) Use the separation of variables technique to reduce the PDE to a pair of ODEs.

(b) Show that if the boundary condition

$$u(0, t) = 0 \text{ for all } t \geq 0$$

is imposed, then there are no nontrivial solutions.

3. (25+1 Points) Solve the initial value problem

$$\begin{aligned}y'' + y' &= f(t) \\ y(0) &= 1 \\ y'(0) &= 0\end{aligned}$$

where

$$f(t) = \begin{cases} 0 & \text{for } 0 \leq t < \pi/2 \\ \sin t & \text{for } t \geq \pi/2 \end{cases}$$

Hint: Recall that the graphs of sine and cosine are horizontal translates of one another.

4. (25+1 Points) Consider the system

$$\begin{aligned}x_1' &= -5x_1 + \alpha x_2 \\x_2' &= -3x_1 + x_2\end{aligned}$$

where α is some positive parameter.

- (a) Would you characterize this as a cooperative system, a parasitic system, or a competitive system? Clearly explain.
- (b) Argue that if $\alpha > 3$, then solution trajectories spiral inward toward the origin.
- (c) Let $\alpha = 10/3$. Give the general solution to the system. Sketch the trajectory which begins at the point $(1, 3)$. (An extra page is provided if needed.)

Extra page for Problem 4