Mathematical Sciences Department
Carnegie Mellon University

Differential Equations
21-260

Fall 2007 Exam 2

No calculator of any kind is permitted. Show all work and give clear explanations.

NAME:

Question	Points	Score	Pres Pt
1	21+1		
2	21+1		
3	21+1		
4	16+1		
5	16+1		
Total	100		

1. (21+1 Points) Solve the initial value problem using Laplace transform methods.

$$y'' + 3y' + 2y = 4,\ y(0) = 0,\ y'(0) = 1$$

2. (21+1 Points) Find the general solution to the differential equation.

$$y'' - 2y' + y = 3\sin 2t - 7\cos 2t$$

3. (21+1 Points) Solve the initial value problem.

$$4y'' + 4y' + 5y = 0,\ y(0) = -2,\ y'(0) = 0$$

4. (16+1 Points) Exactly one of the two families below is the general solution to $4t^2y'' + 8ty' + y = 0$. Which is it? Justify.

   (i) $\dfrac{1}{\sqrt{t}}(A + B\ln t)$

   (ii) $At^2 + \dfrac{B}{\sqrt{t}}$

5. (16+1 Points) A load with mass of 4 units is attached to a spring with spring constant 3. The load is pulled down one inch from its equilibrium position and then is set in motion with an upward strike that imparts an initial speed of one inch per second. Over the next ten seconds, the load oscillates, passing through its equilibrium position several times. Write down an initial value problem consistent with this information. Justify.