Mathematical Sciences Department
Carnegie Mellon University

Differential Equations
21-260

Fall 2007 Exam 1

No calculator of any kind is permitted. Show all work and give clear explanations.

NAME:

Question	Points	Score	Pres Pt
1	24+1		
2	24+1		
3	24+1		
4	24+1		
Total	100		

1. (24+1 Points)

   (a) Solve the initial value problem

$$\begin{aligned} y' &= (1-y)^2 \sin t \\ y(0) &= 1/2 \end{aligned}$$

   and determine the maximal domain (aka "interval of validity") of the solution.

   (b) If the initial condition is changed to $y(0) = 0$, what is the maximal domain?

2. (24+1 Points) Solve the initial value problem

$$\begin{aligned} y' &= e^{4t} + 2y \\ y(0) &= y_0 \end{aligned}$$

and determine whether the maximal domain depends on y_0. If so, state how; if not, explain why not.

3. (24+1 Points)

(a) Determine conditions on (t_0, y_0) for which the initial value problem

$$\begin{aligned} y' &= \ln(y - 1/|t|)^2 \\ y(t_0) &= y_0 \end{aligned}$$

is guaranteed to have a unique local solution.

(b) Could there be any solutions to $y' = \ln(y - 1/|t|)^2$ which are defined on the whole real line, $(-\infty, \infty)$? Explain.

4. (24+1 Points) Suppose that in open waters, with unlimited food supply, a species of fish satisfies the population model $P' = 0.01P$, where $P = P(t)$ is the fish population at time t, measured in weeks. But suppose that when confined to a certain lake, there is only enough food and space to support a population of about 5000 of these fish.

   (a) Write down a differential equation that models population in the lake.

   (b) Suppose the lake is opened to *recreational* fishing. Assume that the more populous are the fish, the easier they are to catch, and the less populous, the harder to catch. Write down a differential equation that models this scenario.

   (c) There must be some proportionality constant in your equation in (b). Would it be reasonable to expect that this constant could be greater than 0.01? Explain.

   (d) Draw a clearly labeled solution portrait corresponding to your equation in (b). Consider only nonnegative t and P. (An extra page is provided if needed.)

Extra page for Problem 4