Review List for the Final Exam

You should know the trigonometric formulas

$$\sin^2\theta + \cos^2\theta = 1$$

and

$$\sec^2\theta = \tan^2\theta + 1,$$

but any other trig formulas you need will be given.

I recommend studying the solutions to exams 1, 2, and 3 as part of your preparation. It is not a bad idea to go back and try to do all of the assigned homework problems as well, and then afterward to study all the homework solutions. (In lieu of working the assigned problems from sections 11.3 through 11.6, though, you could try working a bunch of problems from section 11.7, in which all the different types of series are mixed together; in this way, you can test how well you decide on strategies for establishing convergence or divergence of series.)

Now I highlight some specific topics for you to focus on:

1. Evaluating trigonometric integrals. There were basically two types, $\int \sin^m x \cos^n x dx$ and $\int \sec^m x \tan^n x dx$, but the strategies used to evaluate these integrals may also be applicable to an integral like $\int \cos^m x \tan^n x dx$, for example.

2. Trigonometric substitutions of the form $x = a\sin\theta$, $x = a\tan\theta$, or $x = a\sec\theta$, which may be helpful when the integrand contains an expression of the form $(x^2+a^2)^n$, $(x^2-a^2)^n$, or $(a^2-x^2)^n$. (The power is usually $\frac{1}{2}$, but a trig substitution is often applicable if n is some other power: See example 6 on page 493; also note that a trig substitution can be used to evaluate $\int \frac{1}{x^2+1}dx$ if you forget that $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{x^2+1}$.) When you evaluate integrals via trig substitution, remember to convert your antiderivative into a function of x, and give the function in the most explicit form possible. So if you have
$$\ln|\sec\theta| + 2\theta - \cos 2\theta + C$$
as the antiderivative in terms of θ, and you made the substitution $x = a\sin\theta$, writing
$$\ln|\sec(\sin^{-1}(x/a))| + 2\sin^{-1}(x/a) - \cos(2\sin^{-1}(x/a)) + C$$
is not acceptable as the final form of the antiderivative. The function above is very difficult to understand; it is not clear what the function is "doing" to the variable x. So instead you need to draw a right triangle and use it to find $\sec\theta$ and $\cos 2\theta$ in terms of x. You will find the antiderivative to be
$$\ln\left|\frac{a}{\sqrt{a^2-x^2}}\right| + 2\sin^{-1}(x/a) - \frac{2(a^2-x^2)}{a^2} + C$$
The above function is much easier to understand. (Note that there is nothing we can do to simplify the $\sin^{-1}(x/a)$ term, so we leave that one alone.)

3. Know how to evaluate antiderivatives of rational functions using the partial fraction technique. You should review all the cases, which differ from each other according to how the denominator factors. We considered four cases: all linear factors and none repeated, repeated linear factors, irreducible nonrepeated quadratic factors, and repeated quadratic factors. You should know how to handle any of these, so I recommend working as many 7.4 problems as you can.

4. Know the arc length formula (section 8.1) and how to apply it.

5. Review the techniques in section 7.8 for approximating integrals. These are the Endpoint Methods, the Midpoint Rule, the Trapezoidal Rule, and Simpson's Rule. Be able to draw diagrams to illustrate the methods. If you choose not to memorize the various formulas, then make sure that you can quickly derive them when needed.

6. Review section 7.9 and be able to compute improper definite integrals. Remember that one *always* needs to use limits of proper integrals in order to treat improper integrals and determine whether they exist. *Do not* write meaningless expressions such as

$$\left[-\frac{1}{x^2}\right]_1^\infty, \quad f(\infty), \quad \text{or} \quad \frac{1}{\infty}$$

and then invoke non-laws such as $\frac{1}{\infty} = 0$. The symbol ∞ does not represent a number, that one would substitute into a function to determine a corresponding output. Search through the text and through your lecture notes and you will *never* see the symbol ∞ treated that way. It is only used in limit notation, interval notation, or improper integral notation (where it is really just another form of interval notation). We did not cover the Comparison Theorem for Integrals (bottom of page 536).

7. Know how to recognize the following types of differential equations:

   (a) separable equation
   (b) linear first-order equation
   (c) linear second-order equation (with constant coefficients)

   We have specific strategies for solving differential equations of the types above; these strategies are summarized below:

   (a) If the equation is separable, separate variables and antidifferentiate both sides. Then solve for the unknown function y whenever possible.
   (b) The linear first-order equations we encountered in section 15.2 can be solved by the method of *integrating factors* (see page 633): Multiply both sides of the equation by an appropriate function, so that the left hand side of the new equation is the derivative of a product of functions. Next antidifferentiate both sides and solve for y, if possible.
   (c) We had a "recipe" for solving second-order linear differential equations with constant coefficients; there are three different forms for the general solution, and these were given in a handout in lecture. Recall that that handout showed up as a reference sheet on Exam 2. You will be provided this information again for the Final Exam.

8. Newton's Method (section 4.9) will not be covered on the final exam.

9. Review sequences, including Theorem 3 on page 704, the Squeeze Theorem, the Monotonic Sequence Theorem, recursive sequences, etc. I would re-read section 11.1, reviewing all the text examples. And of course, do quite a few practice problems from the section 11.1 exercises to remind yourself how to analyze various sequences to prove convergence or nonconvergence, and in case of convergence, to find the value of the limit if possible.

10. Review all of the various series tests for convergence and/or divergence, in sections 11.2 through 11.6. Once again, I recommend 11.7 for reading and working problems in order to test your ability to distinguish between various types of series. Know how to recognize a geometric series an alternating series, a p-series, the harmonic series, etc. Make certain that you understand the definition of series convergence (or nonconvergence, as the case may be) on page 714. Given a series, you should be able to write down at least the first few terms of the sequence of partial sums, and possibly determine a formula for the Nth partial sum. In some cases it is easy to see whether that sequence is converging. (Refer to Exercise 1(b) on Exam 3.)

11. On Exam 3, you were not asked any question requiring your knowledge of the Alternating Series Estimation Theorem (page 738) or the Integral Test Estimation Theorem (page 727). You will be asked something concerning at least one of these on the Final Exam.

12. Know how to recognize a power series, and to be able to use whatever test(s) is/are applicable to find the values of x for which the series converges. Understand *radius of convergence* and *interval of convergence*. Know how to give a power series representation of a given function (as in section 11.9), possibly by differentiating or antidifferentiating another power series.

13. Review Taylor and Maclaurin series. Given a function, you should be able to give its Taylor series expansion at any point a. I do not think it is necessary to memorize the various Maclaurin series given at the bottom of page 767, but you should be able to derive any of those Maclaurin series, given the corresponding function. You should also be able to use these Maclaurin series for common functions to derive Maclaurin series for more sophisticated functions such as e^{x^2}, $x\sin(x^3)$, xe^{-x}, and so on.

14. Review Taylor's Inequality (page 763), and know how Taylor polynomials are used to approximate functions (section 11.12).