1. Determine the antiderivative.

$$\int \frac{1}{x^2\sqrt{x^2+4}}dx$$

2. Evaluate

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\ dx$$

3. Consider

$$\int \frac{6x^4 - 2x^3 + 4x^2 - x + 1}{x(2x^2+1)^2}dx$$

   (a) Explain why we would not apply long division in an attempt to simplify the integrand.

   (b) Write out the form of the partial fraction decomposition. **Do not determine the numerical values of the coefficients.**

4. Give a formula for

$$\int \sin^3 x \cos^n x dx$$

   where n is any nonnegative integer.

5. Determine the antiderivative.

$$\int \sin^4 x dx$$

6. Determine the antiderivative.

$$\int \frac{x^3}{\sqrt{x^2+9}}dx$$

7. (a) Use an illustration and explain the Left Endpoint Rule for approximating a proper definite integral $\int_a^b f(x)dx$. Derive the formula for the Left Endpoint Rule.

   (b) Under what condition will the Left Endpoint Rule give an overestimate of $\int_a^b f(x)dx$? Illustrate with a picture.

   (c) Suppose $f(x) > 0$ and $f''(x) < 0$ for all $x \in [a, b]$. Explain why the Trapezoidal Rule will give an underestimate of $\int_a^b f(x)dx$.

8. Evaluate

$$\int_0^{\infty} xe^{-x}dx$$

9. In each part below, evaluate the integral, or demonstrate that the integral does not exist.

   (a) $\displaystyle\int_2^\infty \frac{1}{x^3} dx$

   (b) $\displaystyle\int_{-1}^{8} x^{-2/3} dx$

   (c) $\displaystyle\int_{-\infty}^{\infty} \sin x \; dx$

10. Recall that if f is a twice-differentiable function on an interval $[a, b]$, and n subintervals are used for a Trapezoidal Rule approximation to $\displaystyle\int_a^b f(x)dx$, and f'' is known to be bounded, then the error associated with the approximation is no greater than $\displaystyle\frac{K(b-a)^3}{12n^2}$, where K is any positive number for which $|f''(x)| \leq K$ for all x in $[a, b]$. Determine a number of subintervals which would guarantee accuracy to within $0.001 = \frac{1}{1000}$ in a Trapezoidal Rule approximation to $\displaystyle\int_0^2 \frac{1}{30} \sin(x^2)dx$.

11. Determine the antiderivative: $\displaystyle\int \frac{x^3}{27}\sqrt{9-x^2} \; dx$

12. Determine the antiderivative using the partial fractions technique: $\displaystyle\int \frac{1}{x^2(1-x^2)} \quad dx$

13. Determine whether the integral exists. If so, find its value.

   (a) $\displaystyle\int_1^\infty \frac{1}{(2x+1)^2} \quad dx$

   (b) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sqrt[5]{1-\sin x}} \quad dx$

   (c) $\displaystyle\int_0^{\pi/2} \sec x \;\; dx$

14. (a) Compute the Trapezoidal Rule approximation to $\displaystyle\int_\pi^{4\pi} \sqrt{x}\sin x \;\; dx$ with six subintervals. Simplify the result.

   (b) Recall the error bound theorem for the Trapezoidal Rule: If n subintervals are used to approximate $\displaystyle\int_a^b f(x)dx$ by the Trapezoidal Rule, and if K is any positive number for which $|f''(x)| \leq K$ for all $x \in [a, b]$, then the error in the approximation is no greater than

$\dfrac{K(b-a)^3}{12n^2}$. Show that the error in your approximation in part (a) is no greater than

$$\frac{\pi^3}{16}\left(\frac{1}{4\pi^{3/2}}+\frac{1}{\sqrt{\pi}}+2\sqrt{\pi}\right)$$

15. (a) Clearly explain the distinction between a *proper* integral and an *improper* integral.
    (b) Suppose an improper integral exists. Explain why one must know how to evaluate proper Riemann integrals in order to find the value of the improper integral.
    (c) Explain why some improper integrals do not exist and therefore cannot be assigned a numerical value.

16. Recall the error bound theorem for the Midpoint Rule: If n subintervals are used to approximate $\int_a^b f(x)dx$ by the Midpoint Rule, and if K is any positive number for which $|f''(x)| \le K$ for all $x \in [a, b]$, then the error in the approximation is no greater than $\dfrac{K(b-a)^3}{24n^2}$. Let I_1 and I_2 be the following integrals:

$$I_1 = \int_{-1}^{1} e^{\sin x}dx; \qquad I_2 = \int_{-1}^{1} \sin(e^x)dx$$

Suppose it is desired to find the values of I_1 and I_2 accurate to six decimal places using the Midpoint Rule. Which integral will likely require more subintervals to guarantee the specified accuracy?

17. Determine the antiderivative: $\displaystyle\int \sec^6 x \tan^{10} x \; dx$

18. Determine the antiderivative: $\displaystyle\int \frac{1}{x^2-9}dx$

19. Determine the antiderivative: $\displaystyle\int \frac{10x^4+x^3+90x^2+4x+160}{x(x^2+4)^2} \; dx$

20. Determine the antiderivative: $\displaystyle\int \frac{2x^3-4x^2-48x+252}{x^4-12x^3+36x^2} \; dx$

21. Using one or more illustrations and a verbal argument, explain how the following may occur: A function f is increasing on an interval $[a, b]$, and the Trapezoidal Rule will yield an underestimate to $\int_a^b f(x) \; dx$, no matter how many subintervals of $[a, b]$ are used for the approximation.