Variational Problems in Weighted Sobolev Spaces

Ana Maria Soane
University of Maryland, Baltimore County
asoane@math.umbc.edu

Abstract: We study the Poisson problem $-\Delta u = f$ and Helmholtz problem $-\Delta u + \lambda u = f$ in bounded domains with corners in the plane and $u = 0$ on the boundary. On non-convex domains of this type, the solutions are in the Sobolev space $H1$ but not in $H2$ even though f may be very regular. We formulate these as variational problems in weighted Sobolev spaces and prove existence and uniqueness of solutions in what would be weighted counterparts of $H2 \cap H1_0$. The specific forms of our variational formulations are motivated by, and applied to, a finite element scheme for the time-dependent Navier-Stokes equations.