LYAPUNOV FUNCTIONALS IN STABILITY OF STOCHASTIC DIFFERENCE EQUATIONS

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Preface

Hereditary systems (or systems with delays, or systems with aftereffect) are systems, which future development depends not only on their present state but also on their previous history. Systems of such type are widely used to model processes in physics, mechanics, automatic regulation, economy, biology, ecology etc., (see, e.g., [6-8, 11, 89, 94, 117, 118, 120, 121,128, 149, 185, 258]). An important element in the study of such systems is their stability. As was proposed by Krasovskii [146-148], stability condition for differential equation with delays can be obtained using appropriate Lyapunov functional. By that the construction of different Lyapunov functionals for one differential equation allows to get different stability conditions for the solution of this equation. However the construction of each Lyapunov functional required a unique work from its own author. In 1975, Shaikhet [237] introduced a parametric family of Lyapunov functionals, so that an infinite number of Lyapunov functionals were used simultaneously. This way allowed to get different stability conditions for considered equation using only one Lyapunov functional. At last in the 90's the general method of Lyapunov functionals construction was proposed by Kolmanovskii and Shaikhet for stochastic functional-differential equations and developed later consistently for stochastic difference equations with discrete time and continuous time, for partial differential equations [39, 123-127, 129-138, 174, 178, 190-195, 221-225, 227-229, 231-236, 241, 242]. The method was successfully applied to stability research of some mathematical models in mechanics and biology [20, 27-29, 226, 238-240].

Stability theory for stochastic differential equations (both without delays and with delays) is well studied (see, e.g., [13, 14, 81, 86, 94, 106, 109, 117, 118, 120, 121, 176, 177, 185]). Difference equations, which arise as numerical analogues of differential or integral equations as well as independent mathematical models of dynamical systems with discrete time, have also enjoyed a considerable share of research attention [5-9, 12, 15, 49-53, 62-69, 93, 96, 108, 111, 119, 158, 159, 246, 271].

In this book, consisting of ten chapters, the general method of Lyapunov functionals construction for stochastic difference Volterra equations is expounded.

Introductory Chapter 1 presents basic definitions, Lyapunov type theorems, formal procedure of Lyapunov functionals construction and some useful lemmas.

In Chapter 2 the procedure of Lyapunov functionals construction, described in Chapter 1, is used for p-stability investigation of a simple stochastic difference equation with constant coefficients. It is shown that different ways of Lyapunov functionals construction allow to get different conditions for asymptotic p-stability of the zero solution of this equation.

Chapter 3 generalizes the material from Chapter 2, by applying the procedure of Lyapunov functionals construction to obtain conditions for stability of stochastic difference equation with stationary coefficients. Four different ways of Lyapunov functionals construction are shown. In addition, asymptotic behavior of the solution is studied by characteristic equation.

In Chapter 4 via the procedure of Lyapunov functionals construction different types of sufficient stability conditions are obtained for linear equations with non-stationary coefficients.

In Chapter 5 some important features of the proposed method of Lyapunov functionals construction are considered. In particular, the necessary and sufficient condition for asymptotic mean square stability of the zero solution of stochastic linear difference equation and sufficient stability conditions for the difference equation with Markovian switching are obtained.

In Chapter 6 the general method of Lyapunov functionals construction is used to get asymptotic mean square stability conditions for systems of stochastic linear difference equations with varying delays. Sufficient stability conditions are formulated in terms of existence of positive definite solutions of some matrix Riccati equations.

In Chapter 7 the procedure of Lyapunov functionals construction considered above is applied to different types of nonlinear stochastic difference equations and to different types of stability: asymptotic mean square stability, stability in probability, almost sure stability.

Chapter 8 studies asymptotic behavior of solutions of stochastic difference Volterra equations of the second kind. For that the general method of Lyapunov functionals construction is used as well as the resolvent representation of solutions. In particular, linear and nonlinear equations are considered with constant and with variable coefficients.

In Chapter 9 it is shown that after some modification of the basic Lyapunov type theorems the general method of Lyapunov functionals construction can be applied also for difference equations with continuous time that are enough popular with researches. Some of the previous results are reformulated for such equations and some peculiarities of its investigation are shown.

In Chapter 10 a capability of difference analogues of differential equations to save a property of stability of solutions of considered differential equations is discussed. In particular, sufficient conditions on discretization step, at which a property of stability is saved, are obtained for some known mathematical models: inverted pendulum, Nicholson's blowflies equation, predator-prey model, for integro-differential equation of convolution type.

The book is mostly based on the results obtained by the author independently or jointly with coauthors, in particular, with the friend and colleague V.Kolmanovskii, with whom the author is glad and happy to collaborate for more than 30 years. The bibliography at the end of the book does not pretend to be complete and includes some of the author's publications [221-242], his publications jointly with coauthors [11, 20, 27-29, 39, 60, 80, 116, 123-138, 145, 150, 174, 178, 190-195, 210, 243-245] as well as the literature used by the author during his preparation of this book.

Note that all objects (equations, theorems, lemmas and so on) in the book have a two number reference. However, when referring to an object from another chapter, a third number, indicating the corresponding chapter, is added in front.

The book is addressed both to experts in stability theory as well as to a wider audience of professionals and students in pure and computational mathematics, physics, engineering, biology and so on.

The author will appreciate receiving useful remarks, comments and suggestions.