

## MATH 21-373 - ALGEBRAIC STRUCTURES

### Homework Assignment 3

- (1) 2.5.2
- (2) 2.5.3
- (3) 2.5.12
- (4) 2.5.13
- (5) 2.5.14
- (6) 2.5.15
- (7) 2.5.25
- (8) Prove that  $GL_n(F)$  is non-abelian for any  $n \geq 2$  and any field  $F$ .
- (9) Let  $G$  be an abelian group. Prove that  $\{g \in G \mid o(g) < \infty\}$  is a subgroup of  $G$  (called the torsion subgroup of  $G$ ). Give an explicit example where this set is not a subgroup when  $G$  is non-abelian.
- (10) A group  $G$  is called finitely generated if there is a finite set  $A$  such that  $G = \langle A \rangle$ .
  - (a) Prove that every finitely generated subgroup of the additive group  $\mathbb{Q}$  is cyclic.
  - (b) Prove that  $\mathbb{Q}$  is not finitely generated.