MATH 21-373 - ALGEBRAIC STRUCTURES

Homework Assignment 10

(1) 2.14.4d

(2) Which of the following abelian groups are isomorphic? Here, $\{a_1, \ldots, a_k\}$ denotes $\mathbb{Z}/a_1\mathbb{Z} \times \cdots \times \mathbb{Z}/a_k\mathbb{Z}$.
   - $\{5^2 \cdot 7^2, 3^2 \cdot 5 \cdot 7\}$
   - $\{3^2 \cdot 5^2 \cdot 7, 5 \cdot 7^2\}$
   - $\{3 \cdot 5^2, 7^2, 3 \cdot 5 \cdot 7\}$
   - $\{5^2 \cdot 7, 3^2 \cdot 5 \cdot 7^2\}$

(3) Find all finite groups which have exactly two conjugacy classes.

(4) Prove that a subgroup of index two is necessarily normal.

(5) How many elements of order 7 must there be in a simple group of order 168?

(6) Prove that there are no non-abelian simple groups of order less than 60.

   Remark 1: Note, that you may have to use different arguments depending on the order of the group. You are free to use any results proven in class or in the homework assignments.

   Remark 2: The number of points for this problem will be three times the usual.

(7) (Extra credit) Prove, that the only non-abelian simple group of order ≤ 100 is the alternating group A_5, which, recall, has order 60.