Math 371 - Lie Theory

Homework Assignment 2
Due Sep 7

1. Prove that if F is a field then $\forall a, b \in F$ if $a \neq 0, b \neq 0$ then $ab \neq 0$.

2. Let $\mathbb{H}$ be the space of quaternions defined as follows. $\mathbb{H} = \{a + \mathfrak{i}b + \mathfrak{j}c + \mathfrak{k}d\}$ with componentwise addition and multiplication defined as

$$\mathfrak{i}^2 = \mathfrak{j}^2 = \mathfrak{k}^2 = -1, \mathfrak{i}\mathfrak{j} = \mathfrak{k}, \mathfrak{j}\mathfrak{k} = \mathfrak{i}, \mathfrak{k}\mathfrak{i} = \mathfrak{j}$$

   and extended $\mathbb{R}$-linearly.

   Show that the map from $\mathbb{H}$ to the set of 2×2 matrices of the form

$$\begin{pmatrix} a + \mathfrak{i}d & -b - \mathfrak{i}c \\ b - \mathfrak{i}c & a - \mathfrak{i}d \end{pmatrix}$$

   is an isomorphism of fields, if the field operations on the image are regular matrix addition and multiplication.

3. Problem 1.4.1 from the textbook.

4. Problem 1.4.2 from the textbook.

5. Problem 1.4.3 from the textbook.

6. Problem 1.4.4 from the textbook.