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Asymptotic behaviour and correctors for linear Dirichlet pbs with simultaneously varying operators and domains.

$$\begin{cases} u^\varepsilon \in H^1_0(\Omega_\varepsilon) \\ -\operatorname{div}(A^\varepsilon D u^\varepsilon) = f \text{ in } \mathcal{D}'(\Omega_\varepsilon) \end{cases}$$

$A^\varepsilon \in M_\alpha^\beta$: $A^\varepsilon \geq \alpha I$, $A^\varepsilon \leq \beta I$ $\quad A^\varepsilon \xrightarrow{H} A^0 \quad \{\Omega_\varepsilon\} \subseteq \Omega$

$$\mu_\varepsilon(B) = \begin{cases} 0 & \text{if } \operatorname{Cap}(B \setminus \Omega_\varepsilon) = 0. \\ +\infty & \text{otherwise} \end{cases}$$

$x_n > 0$

$x_n \leq 0$

$v \in H^1_0(\Omega) \cap L^2(\Omega, \mu_\varepsilon) \Rightarrow v = 0$ on $\Omega \setminus \Omega_\varepsilon$ q.e. $v \in H^1_0(\Omega_\varepsilon)$.

$\mu_\varepsilon \in \mathcal{M}(\Omega)$.

$$(*) \begin{cases} u^\varepsilon \in H^1_0(\Omega) \cap L^2(\Omega, \mu_\varepsilon) \\ \int_\Omega A^\varepsilon D u^\varepsilon D y \, dx + \int_\Omega u^\varepsilon y \, d\mu_\varepsilon = \int_\Omega f y \, dx \end{cases}$$

$\forall y \in H^1_0(\Omega) \cap L^2(\mu_\varepsilon)$.

$\bar{A}^\varepsilon$ = adjoint of A^ε

$\{\mu_\varepsilon\}_{\varepsilon > 0}$. Suppose you can produce slns w^ε to

$$\begin{cases} w^\varepsilon \in H_0^1(\Omega) \cap L^2(\Omega;\mu_\varepsilon) \\ \int_\Omega \overline{A}^\varepsilon Dw^\varepsilon Dy\,dx + \int_\Omega w^\varepsilon y\,d\mu_\varepsilon = \int_\Omega y\,dx \end{cases}$$

and assume also that $w^\varepsilon \geq 0$ q.e.

$w^\varepsilon \xrightarrow{H_0^1} w^0 \Rightarrow u^\varepsilon \to u^0$ sln to $(*)$ w/ $\varepsilon = 0$

$y_1 = w^\varepsilon \varphi$, $y_2 = u^\varepsilon \varphi$

$$\int_\Omega (\overline{A}^\varepsilon Dw^\varepsilon D\varphi u^\varepsilon - A^\varepsilon Du^\varepsilon D\varphi w^\varepsilon)\,dx = \int_\Omega \varphi(u^\varepsilon - w^\varepsilon f)\,dx$$

$u_\varepsilon \to u$ H_0^1 + $w^\varepsilon \to w^0$ H_0^1

$A^\varepsilon Du^\varepsilon \to A^0 Du$, $\overline{A}^\varepsilon Dw^\varepsilon \to \overline{A}^0 Dw^0$.

$$\int_\Omega (\overline{A}^0 Dw^0 D\varphi u - A^0 Du D\varphi w^0)\,dx = \int_\Omega \varphi(u - w^0 f)\,dx$$

$$\int_\Omega (\overline{A}^0 Dw^0 D\varphi u^0 - A^0 Du^0 D\varphi w^0)\,dx = \int_\Omega \varphi(u^0 - w^0 f)\,dx.$$

Final step: if v solves

$$\int_\Omega \overline{A}^0 Dw^0 D\varphi v - A^0 Dv D\varphi w^0 = \int_\Omega \varphi v\,dx$$

$\Rightarrow v = 0$.

This will give uniqueness $\Rightarrow u = u^0$.

Plug in $\varphi = v \Rightarrow \int \frac{v^2}{2} - \int w^0 \frac{v^2}{2} \, d\mu_0 - \int_\Omega A^0 D v D w^0 = \int v^2 dx$

$\Rightarrow v = 0.$

The construction of μ_0.

$$\int_\Omega A Dw Dy \, dx + \int_\Omega w y \, d\mu = \int_\Omega y \, d\lambda$$

$$\langle -\mathrm{div}(ADw), y \rangle + \langle w\mu, y \rangle = \langle \lambda, y \rangle$$

$$\langle w\mu, y \rangle = \langle \underbrace{\lambda + \mathrm{div}(ADw)}_{\nu}, y \rangle$$

$$\mu(B) = \begin{cases} \int_B \frac{d\nu}{w} & \text{if } \mathrm{Cap}(B \cap \{w = 0\}) = 0. \\ +\infty & \text{otherwise} \end{cases}$$

We have $\{\mu_\varepsilon\}_{\varepsilon \geq 0}$ assigned. Let w^ε be a sln.

$$\int_\Omega \bar{A}^\varepsilon Dw^\varepsilon Dy \, dx + \int_\Omega w^\varepsilon y \, d\mu_\varepsilon = \int_\Omega y \, d\lambda^\varepsilon.$$

$w^\varepsilon \longrightarrow w^0$ in H^1_0

$\mu_0(B) = \{ \ldots$ as above

$\Rightarrow \int_\Omega \bar{A}^0 Dw^0 Dy \, dx \ldots$