Department of Mathematical Science
Carnegie Mellon University

21-272 - Introduction to PDEs

Spring 2017

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Classes: classes will be on MWF 11:30-12:20 in BH 235A
Office hours: Thursday 5:30-6:30

Suggested textbook: Strauss - *Partial differential equations, an introduction*

Prerequisite: A good knowledge of ODEs and multi-variable calculus is required.
A list of suggested courses with the required prerequisite is the following:

- ODEs: 21-260 or 21-261
- multi-variable calculus: 21-259, 21-268 or 21-269

or permission of the instructor. The main tools needed will be reviewed along the way.

Aim: the aim of the course is to give a broad overview of the most important first and second order linear PDEs (namely, transport equation, Laplace equation, heat equation and wave equation) as well as of some non-linear ones. In particular, Fourier series and Fourier transform will be introduced and applied to study some linear PDEs.

Time permitting, some topics from first order non-linear PDEs will be covered.

Syllabus: The following topics will be covered for sure:

(1) Introduction to the course: motivations and brief history of PDEs
(2) Some technical preliminaries
(3) The transport equation and first order linear PDEs
  - constant coefficients
  - variable coefficients - the characteristic method
  - inhomogeneous equation - Duhamel's principle
  - example: the traffic flow
  - example: the Burger's equation

(4) The one dimensional diffusion equation
  - derivation from random walks and boundary conditions
  - diffusion with drift and telegrapher equation
  - maximum principle
  - the homogeneous global Cauchy problem - the heat kernel
  - the inhomogeneous global Cauchy problem - Duhamel's principle
  - diffusion in bounded domains - separation of variables

(5) The one dimensional wave equation
- derivation and boundary conditions
- the homogeneous global Cauchy problem - d'Alambert's formula
- the inhomogeneous global Cauchy problem - Duhamel's principle
- causality, energy and uniqueness
- wave equation in bounded domains - separation of variables and reflection

(6) Fourier series
- definition and properties
- how to compute the Fourier coefficients
- convergence theorems
- solutions for the heat and the wave equation in bounded domains
- inhomogeneous boundary conditions

(7) The Laplace equation (*i.e.*, harmonic functions) in 2 and 3 dimensions
- maximum principle
- uniqueness
- harmonic functions in the circle: Poisson's formula
- harmonic functions in special domains
- Dirichlet's principle

(8) The Poisson equation
- first Green's identity and compatibility of the data
- second Green's identity and representation formula
- Green's function
- Green's function in the half-space
- Green's function in the sphere

The rest of the course will be devoted to (some of) the following topics, according to the time we will have left and to the interest of the students.

(9) The heat equation (*i.e.*, the diffusion equation) in 2 and 3 dimensions
- derivation
- uniqueness and backward uniqueness
- stability
- parabolic maximum principle
- fundamental solution (the heat kernel)
- non-homogeneous problem (Duhamel's principle)
- general initial conditions

(10) Eigenfunctions of the Laplacian

(11) The wave equation in 2 and 3 dimensions
- derivation
- domain of influence
- Kirchhoff's formula
- Huygens's principle
- non-homogeneous problem
- example: vibration of a drumhead
- example: solid vibration in a ball (spherical harmonics)

(12) Fourier transform
- definition and properties
- inversion
- application to some linear PDEs

(13) Examples:
- an example from physics: the hydrogen atom

Homework:
- posted on Blackboard on Friday after class every week
- due the week after on Friday at the beginning of the class
- written in **pen** or Latex
- collaboration among students is encouraged, but each one has to present her/his own write up (if hand-written, please write it in a comprehensible way!).
- sometimes solutions of the previous week will be presented by students (this will help you in discussing the problems in order to understand them better!)
- late homework: ask permission for a good reason

Exams:
- two midterms on regular class hours (mid of February - begin of April)
- a 3-hour in class final during the final week (beginning of May)

All midterms and exam are with **closed** books and **no calculators/computers/...**.
Everything written in **pen**.
Part of the final exam will be made by homework problems.

Grades:
- 30% homework (equally evaluated - the lowest homework grade will be dropped)
- 30% the best midterm
- 40% final