21470 - Topic in Analysis

Calculus of Variations

Spring 2016

Syllabus

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Office Hours: to be announced on Blackboard
Course webpage: http://www.math.cmu.edu/~rcristof/teaching/spring16.html

Classes: classes will be on M W F 9:30-10:20 in WeH 8427

Aim: The aim of the course is to provide the students the basic concepts and tools of the classical theory of the Calculus of Variations. Solutions of some classical minimization problems, as well as relevant historical notes, will also be presented along the way.

Program:
- introduction to the course: motivations and brief history of CoV
- some classical problems: brachistochrone, surfaces of revolution, isoperimetric problem, hanging cable
- technical preliminaries
- C^2 theory: first and second Euler-Lagrange equations
- C^1 theory: first and second E.-L. equations (and a regularity result)
- broken extremals: first and second Weierstrass-Erdmann corner condition
- constraint minimization, *i.e.*, Lagrange multipliers theory
- free boundary problems
- suffcient conditions: convexity, Jacobi conjugate points, Weierstrass field theory
- solution of classical problems (brachistochrone, surface of revolution)
- Lavrentiev's phenomenon
- some possible extensions (depending on time and on student's interest!):
  - problems with vector-valued functions
  - problems with high-order derivatives

Notes: No textbook needed; notes will be provided during the course.

Homework: posted on Blackboard on Friday after class every two weeks; due the week after on Friday at the beginning of the class.
You are allow one late homework (*i.e.*, one week more).
No other late homeworks are allowed!
Collaboration among students is encouraged, but each one has to present her/his own write up.

Exams: one midterm (March 2nd) on regular class hours.
A 3-hour in class final during the final week (beginning of May).

Grades: 1/3 homeworks (equally evaluated), 1/3 midterm, 1/3 final.
The lowest homework grade will be dropped.