

# Putnam E.07

Po-Shen Loh

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## 1 Problems

**Putnam 2018/B1.** Let  $\mathcal{P}$  be the set of vectors defined by

$$\mathcal{P} = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \mid 0 \leq a \leq 2, 0 \leq b \leq 100, \text{ and } a, b \in \mathbb{Z} \right\}.$$

Find all  $\mathbf{v} \in \mathcal{P}$  such that the set  $\mathcal{P} \setminus \{\mathbf{v}\}$  obtained by omitting vector  $\mathbf{v}$  from  $\mathcal{P}$  can be partitioned into two sets of equal size and equal sum.

**Putnam 2018/B2.** Let  $n$  be a positive integer, and let  $f_n(z) = n + (n-1)z + (n-2)z^2 + \cdots + z^{n-1}$ . Prove that  $f_n$  has no roots in the closed unit disk  $\{z \in \mathbb{C} : |z| \leq 1\}$ .

**Putnam 2018/B3.** Find all positive integers  $n < 10^{100}$  for which simultaneously  $n$  divides  $2^n$ ,  $n-1$  divides  $2^n - 1$ , and  $n-2$  divides  $2^n - 2$ .