

Putnam E.06

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1 Problems

Note: Prof. Loh is on the way back in from the airport, and will arrive at 5:10pm. Go ahead and start working on the problems anyway. The solutions will be discussed at the usual time.

Putnam 2018/A1. Find all ordered pairs (a, b) of positive integers for which

$$\frac{1}{a} + \frac{1}{b} = \frac{3}{2018}.$$

Putnam 2018/A2. Let $S_1, S_2, \dots, S_{2^n-1}$ be the nonempty subsets of $\{1, 2, \dots, n\}$ in some order, and let M be the $(2^n - 1) \times (2^n - 1)$ matrix whose (i, j) entry is

$$m_{ij} = \begin{cases} 0 & \text{if } S_i \cap S_j = \emptyset; \\ 1 & \text{otherwise.} \end{cases}$$

Calculate the determinant of M .

Putnam 2018/A3. Determine the greatest possible value of $\sum_{i=1}^{10} \cos(3x_i)$ for real numbers x_1, x_2, \dots, x_{10} satisfying $\sum_{i=1}^{10} \cos(x_i) = 0$.