Homework Set 5

1) Let A be a nonzero hermitian matrix. Show that $tr(AA^*) > 0$.

2 a) Show that the absolute value of the determinant of a real unitary matrix is 1.
b) If A is a complex square matrix, then show that $Det(\overline{A}) = \overline{Det(A)}$. Conclude that the absolute value of the determinant of a complex unitary matrix is 1.

3) Let $A : V \to V$ be a symmetric linear map. Show that the index of nullity of the form

$$(v, w) \to \langle Av, w \rangle$$

is equal to the dimension of the kernel of A. Show that the index of positivity is equal to the number of eigenvectors in a spectral basis having a positive eigenvalue.

4) If A and B are submodules of M, then show that:
(a) $A \cap B$ is a submodule of M
(b) $A + B = \{a + b : a \in A, b \in B\}$ is a submodule of M
(c) $(A + B)/B$ is isomorphic to $A/(A \cap B)$.