Math 301: Homework 8

Due Wednesday November 14 at noon on Canvas

- 1. Compute the spectrum of K_n and $K_{m,n}$.
- 2. Two vertices in a graph G are called *twins* if they have exactly the same neighborhood. Show that if a graph has a pair of twins then 0 is one of its eigenvalues.
- 3. Let G be a graph and let H be the graph obtained by adding an isolated vertex to G. Show that $\operatorname{spec}(H) = \operatorname{spec}(G) \cup \{0\}.$
- 4. Does the spectrum of a graph determine connectivity? ie, if I tell you the eigenvalues of a graph, can you tell me if it is connected? If not, can you construct two graphs with the same set of eigenvalues, one of which is connected and one of which is disconnected? Problems 1 and 3 may be useful.