Math 301: Homework 3

Due Friday September 21 at noon on Canvas

1. Decide whether the following statements are true or false. If true give a proof, and if false give a counterexample.

   (a) If G is connected and $e \in E(G)$, then G has a spanning tree containing e.

   (b) If G is connected and e, f are distinct edges in G, then G has a spanning tree containing e and f.

   (c) If G is connected and e, f, g are distinct edges in G, then G has a spanning tree containing e, f, and g.

   (d) If G is connected and F is a subset of edges in G which is cycle free, then G has a spanning tree containing all of the edges of F.

2. Let G be a graph. Show that G has a subgraph H where H is bipartite and for each $v \in V(G)$ the degree of v in H is at least half of the degree of v in G.

3. Let G be a graph on n vertices with nd edges. Show that there is a subgraph H of minimum degree $d + 1$.

4. Show that no matter how K_n is embedded in the plane there must be $\Omega(n^4)$ crossings.