Algebra and Calculus facts

Necessities for Amath 351

Note: In what follows, a, b, c, d, α and β denote arbitrary constants; u and v denote arbitrary functions.

1 Special values, limits

- $\sin(0) = 0, \cos(0) = 1,$
  $\sin(\frac{\pi}{2}) = 1, \cos(\frac{\pi}{2}) = 0$
  $\sin(\pi) = 0, \cos(\pi) = -1$
- $\ln(1) = 0$
- $e^0 = 1$
- $\lim_{x \to 0+} \ln(x) = -\infty$
- $\lim_{x \to \infty} e^x = \infty, \lim_{x \to -\infty} e^x = 0$
- $\lim_{x \to 0} \dfrac{\sin(x)}{x} = 1$

2 Algebra

2.1 Elementary algebra

- $a^2 - b^2 = (a - b)(a + b)$
- $(a \pm b) = a^2 \pm 2ab + b^2$
- $\dfrac{a/b}{c/d} = \dfrac{ad}{bc}$
- $ax^2 + bx + c = 0$
  $\Rightarrow x_\pm = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2.2 Exponent rules

- $(a^\alpha)^\beta = a^{\alpha\beta}$
- $a^\alpha a^\beta = a^{\alpha+\beta}$
- $a^{-1} = \dfrac{1}{a}$

2.3 Logarithm rules

- $\ln(ab) = \ln(a) + \ln(b)$
- $\ln(a^\alpha) = \alpha \ln(a)$
- $\ln(e^x) = x, \quad e^{\ln(x)} = x$

2.4 trigonometry

- $\cos^2(a) + \sin^2(a) = 1$
- $\tan(a) = \dfrac{\sin(a)}{\cos(a)}, \quad \cot(a) = \dfrac{\cos(a)}{\sin(a)}$
- $\sin(-a) = -\sin(a), \quad \cos(-a) = \cos(a)$
- $\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$
- $\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

3 Calculus: derivatives

3.1 General rules

- $(au + bv)' = au' + bv'$
- $(uv)' = u'v + uv'$ (product rule)
- $[u(v(x))]' = u'(v(x))v'(x)$ (chain rule)

3.2 Basic derivatives

- $a' = 0$
- $(x^\alpha)' = \alpha x^{\alpha-1}$ (Power rule)
- $(\ln x)' = \frac{1}{x}$
- $(e^{\alpha x})' = \alpha e^{\alpha x}$
- $(\sin(x))' = \cos(x)$, $(\cos(x))' = -\sin(x)$
- $(\arctan(x))' = \frac{1}{1+x^2}$
- $(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$

4 Calculus: integrals

4.1 General rules

- $\frac{d}{dx}\int f dx = f(x), \quad \int f' dx = f(x) + c$
- $\int (au + bv)dx = a\int udx + b\int vdx$
- $\int udv = uv - \int vdu$ (Integration by parts)
- $\int v(u(x))u'(x)dx = \int v(u)du$, $u = u(x)$, u-substitution

4.2 Basic Integrals

- $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + c, \quad \alpha \neq -1,$ Power rule
- $\int \frac{1}{x}dx = \ln(x) + c$
- $\int e^x dx = e^x + c$
- $\int \sin(x)dx = -\cos(x) + c$
- $\int \cos(x)dx = \sin(x) + c$
- $\int \frac{1}{1+x^2}dx = \arctan(x) + c$
- $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x) + c$