MATH251.

Some, but not all, facts from Calculus you need to know.

1. Integration by parts

$$\int udv = uv - \int vdu$$

2. $$\int (ax+b)^n dx = \begin{cases} \dfrac{(ax+b)^{n+1}}{a(n+1)} + C, & n \neq -1 \\ \dfrac{1}{a}\ln|ax+b| + C, & n = -1 \end{cases}$$

3. $$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

4. $$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

5. $$\int \sin axdx = -\frac{1}{a}\cos ax + C$$

6. $$\int \frac{1}{\sin^2 ax}dx = -\frac{1}{a}\cot ax + C$$

7. $$\int \cos axdx = \frac{1}{a}\sin ax + C$$

8. $$\int \frac{1}{\cos^2 ax}dx = \frac{1}{a}\tan ax + C$$

9. $$2\sin^2 ax = 1 - \cos 2ax \Rightarrow \int \sin^2 axdx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

10. $$2\cos^2 ax = 1 + \cos 2ax \Rightarrow \int \cos^2 axdx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

11. $$\int e^{ax}dx = \frac{1}{a}e^{ax} + C$$

12. $$\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}, a > 0$$

13. $$\lim_{x\to\infty}\frac{ax+b}{e^x} = 0, a, b$$ **-any,** i.e. exponent grows faster than linear function

14. $$\lim_{x\to\infty}\frac{x^k}{e^x} = 0$$ **,** for any k **,** i.e. exponent grows faster than any polynomial function

15. $$\lim_{x\to\infty}\frac{\ln x}{x} = 0$$ **,** i.e. logarithmic function is slower than linear

16. $$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

17. $$\cos x = 1 - \frac{x^2}{2!} + \ldots + (-1)^n\frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

18. $$\sin x = x - \frac{x^3}{3!} + \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

19. $$\sum_{n=0}^{\infty}x^n = \frac{1}{1-x}, |x| < 1$$

20. $$\lim_{x\to 0+}(1+x)^{1/x} = e$$