LAPLACE TRANSFORM AND TRANSLATIONS THEOREMS

1. **Definition of the Laplace transform:** $L(f) = \int_0^\infty e^{-ts} f(t)dt$

Denote $L(f) = F(s)$

2. **Definition of the step function :** $u_c(t) = u(t-c) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$

3. **First translation Theorem (Direct form)**

$$L(u_c(t)f(t-c)) = L(u(t-c)f(t-c)) = e^{-cs}L(f(t)) = e^{-cs}F(s)$$

4. **First translation Theorem (Inverse form)**

$$u_c(t)f(t-c) = u(t-c)f(t-c) = L^{-1}(e^{-cs}F(s))$$

5. **Second translation Theorem (Direct form)**

$$L(e^{ct}f(t)) = F(s-c) = L(f(t))|_{s \to s-c}$$

6. **Second translation Theorem (Inverse form)**

$$L^{-1}(F(s-c)) = L^{-1}(F(s)|_{s \to s-c}) = e^{at}f(t)$$