

Putnam $\Sigma.2$

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1 Problems

Putnam 1998/A4. Let $A_1 = 0$ and $A_2 = 1$. For $n > 2$, the number A_n is defined by concatenating the decimal expansions of A_{n-1} and A_{n-2} from left to right. For example $A_3 = A_2A_1 = 10$, $A_4 = A_3A_2 = 101$, $A_5 = A_4A_3 = 10110$, and so forth. Determine all n such that 11 divides A_n .

Putnam 1998/A5. Let \mathcal{F} be a finite collection of open discs in \mathbb{R}^2 whose union contains a set $E \subseteq \mathbb{R}^2$. Show that there is a pairwise disjoint subcollection D_1, \dots, D_n in \mathcal{F} such that

$$E \subseteq \bigcup_{j=1}^n 3D_j.$$

Here, if D is the disc of radius r and center P , then $3D$ is the disc of radius $3r$ and center P .

Putnam 1998/A6. Let A, B, C denote distinct points with integer coordinates in \mathbb{R}^2 . Prove that if

$$(|AB| + |BC|)^2 < 8 \cdot [ABC] + 1$$

then A, B, C are three vertices of a square. Here $|XY|$ is the length of segment XY and $[ABC]$ is the area of triangle ABC .