

# Putnam E.4

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## 1 Problems

**Putnam 2007/A1.** Find all values of  $\alpha$  for which the curves  $y = \alpha x^2 + \alpha x + \frac{1}{24}$  and  $x = \alpha y^2 + \alpha y + \frac{1}{24}$  are tangent to each other.

**Putnam 2007/A2.** Find the least possible area of a convex set in the plane that intersects both branches of the hyperbola  $xy = 1$  and both branches of the hyperbola  $xy = -1$ . (A set  $S$  in the plane is called *convex* if for any two points in  $S$  the line segment connecting them is contained in  $S$ .)

**Putnam 2007/A3.** Let  $k$  be a positive integer. Suppose that the integers  $1, 2, 3, \dots, 3k + 1$  are written down in random order. What is the probability that at no time during this process, the sum of the integers that have been written up to that time is a positive integer divisible by 3? Your answer should be in closed form, but may include factorials.