Teaching Portfolio

Jonathan Simone

August 20, 2020

Contents

1 Overview

I have a diverse range of teaching and mentoring experience, which positions me well to teach across all levels of instruction and to address the needs of various student groups. I have taught coordinated courses, uncoordinated courses, non-credit bearing workshops, and have worked with undergraduates on independent study projects. I have developed curricula, employed various teaching techniques, and implemented technology and active-learning techniques in my classroom. Most recently, I developed the curriculum for my Advanced Calculus and Complex Analysis courses. As a graduate student at the University of Virginia, I developed material for and ran an experimental flipped Calculus I class. See Section 2, Section 3, and Section 4, for more details.

In the Spring of 2020, instructors country-wide were tasked with shifting classes to a remote setting, amid the coronavirus pandemic. For my part, I successfully transitioned from in-class instruction to holding live lectures and office hours online via Zoom so that I could continue to interact with my students in real time. See Section 3.f for more details and Section 6.a.1 for student comments on end-of-semester evaluations regarding my transition from in-class to online instruction. Although the transition was fairly successful, my students admitted there was a lack of motivation to attend virtual classes. To address this, I collaborated with other section instructors to create a “flipped” virtual class (see Section 3.f for details) to encourage student participation and bring back the social component of the college experience.

My dedication to mathematics education goes beyond higher-education. I believe that a stigmatization of mathematics occurs for many students at an early age. Thus, it is important to expose young students to fun, interactive mathematics activities not in the standard curriculum that stimulate their mathematical curiosity. To this end, as a graduate student, I was involved in community outreach programs through the University of Virginia. As a part of these programs, I helped develop and facilitate activities for local elementary school students. See Section 9 for more details.

1.a Awards and Recognition

My commitment to teaching has been recognized by consistent positive student evaluations (see Section 5 and Section 6) and department- and university-wide teaching awards. Most recently, I was a Distinguished Teaching Award finalist at the University of Massachusetts Amherst. At the University of Virginia, I was awarded the All University Graduate Teaching Award. Moreover, as a graduate student, I was invited to run workshop and seminar discussions and I was a mentor for first-time graduate student instructors in the Mathematics Department. See Section 8 for more details.

2 Teaching Experience

In the following list of courses, I was the instructor of record.

University of Massachusetts Amherst, Amherst, MA (2018-present)

- Calculus III
- Introduction to Linear Algebra
- Advanced Multivariable Calculus (visit the class webpage here)
- Complex Variables (visit the class webpage here)
- Theory of Surfaces (1-credit independent study; details in Section 4)
- Theory of Manifolds (3-credit independent study; details in Section 4)
- Knot Theory (Senior Thesis; details in Section 4)

University of Virginia, Charlottesville, VA (2012-2018)

- Calculus I
- Calculus I (flipped classroom setting; see a short video here and details in Section 3.d)
- Applied Calculus I
- Calculus II
- Applied Calculus II
- Calculus III
- Financial Mathematics

John Jay College of Criminal Justice, New York, NY (2010-2012)

- College Algebra
- Finite Mathematics
- Pre-Calculus
- Calculus I

The Higher Education Opportunity Programs are New York State-sponsored programs that provide financial aid and college preparation to underserved and underrepresented students. As a part of these programs, I ran non-credit bearing summer workshops that prepared incoming first-year students for their first semester mathematics courses and semester-long workshops that reinforced student learning throughout each semester.

Opportunity Programs at New York University, New York, NY (2010-2014)

- Calculus I Workshop
- Quantitative Reasoning Workshop (topics include combinatorics, probability, statistics, and applications of exponential growth and decay)

Opportunity Programs at Barnard College, New York, NY (2014)

- Pre-Calculus

3 Curriculum Development

I have taught a number of uncoordinated courses that required me to develop my own teaching materials, classroom structure, and, in some cases, my own curricula. When teaching coordinated courses, on the other hand, I work with the other instructors to decide which topics will be covered and to create exams. Most notably, at the University of Virginia, I was part of a team that developed and refined material for and ran experimental flipped Calculus I classes (see Section 3.d), and with the switch to remote instruction amid the coronavirus pandemic, I am currently working with other Linear Algebra instructors at the University Massachusetts Amherst in creating flipped virtual Linear Algebra classes (see Section 3.f).

3.a Uncoordinated Courses

I have taught many uncoordinated classes—Financial Mathematics, Calculus III, Advanced Multivariable Calculus, and Complex Variables. In these classes, I chose the textbook, decided on the topics I would cover, created homework assignments, and wrote and graded the exams. In higher level courses, I incorporate my research interests, when applicable. For example, in Advanced Multivariable Calculus, I spent time developing the tools of differential forms and used them to discuss the "classical" vector calculus theorems, and in Complex Variables, I used the notion of branch cuts to briefly segue into Riemann surfaces. In Section 7.a, you can find my syllabus for Advanced Multivariable Calculus and in Section 7.c, you can find the final exam I wrote for Complex Variables.

3.b Class Prep Assignments

Some classes benefit from short assignments to be completed prior to class meetings. Such assignments are essential in the flipped classroom setting (see Section 3.d and Section 3.f), but I have found them to also be useful in certain lecture-based courses. In particular, since my Advanced Multivariable Calculus course relied heavily on Calculus III and Linear Algebra, I assigned "pre-class assignments" before every class meeting. These assignments were meant to review pre-requisite material so that students would arrive to class ready for the day's lecture. See Section 7.b for a selection of pre-class problems that I assigned. You may also visit the course webpage to see more details on how the class was run, the full list of pre-class assignments (with solutions), homework assignments, and exams.

3.c Working with Teaching Assistants

Calculus I, Calculus II, and Calculus III at the University of Massachusetts Amherst and the University of Virginia have fourth-hour discussion sections that are run by teaching assistants. These TAs are typically graduate students in the Mathematics Department who have little-to-no teaching experience. Thus, when I teach such a course, I work closely with my TA and act as a mentor and teaching resource. An important aspect of this TA position is to help first-year graduate students develop the skills of writing and grading assignments. Thus I give them the responsibility of writing, administering, and grading short quizzes every week.

In a typical discussion section, I have my TA break the class into groups so that the students can collaborate on worksheets, while the TA walks around the room and answers problems. The purpose of these worksheets is to solidify what the students learned during the week in lecture and to further prepare them for the quiz at the end of the discussion section. In my Calculus III class at the University of Virginia, I also used this opportunity to assign problems that explored applications of the concepts we covered during the week. See Section 7.d for a worksheet that discusses the notion of *torque* in relation to the cross product.

3.d Flipped Calculus I at the University of Virginia

At the University of Virginia, I was part of a team that developed and refined material for and ran experimental flipped Calculus I classes. This team included three other graduate student instructors, the Mathematics Department Director of Lower Division Courses Paul Bourdon, and Gail Hunger from the Learning Design and Technology team at the University of Virginia. The course was structured as follows.

There were three class meetings a week—two 75-minute class meetings and a final 50-minute class meeting. During the two 75-minute class meetings, lecturing was minimal and students engaged in group work and student presentations for a majority of the class. This setting allowed students to learn from each other, learn how to communicate mathematically, and use collaboration as a tool to solve problems that are more challenging than the problems covered during a conventional lecture. During these classes, I circulated throughout the room, acting as an idea catalyst and group work facilitator. To prepare for these class meetings, students watched a short video that introduces new material and then complete 4-5 problems based on the video on webassign.net. Visit `https://youtu.be/r-Pc20qxEh8` to watch a class prep video that I created. See Section 7.b for a screen shot of some of the corresponding webassign problems. These assignments were due 30 minutes before each 75-minute class meeting.

At the beginning of each class meeting, I typically reviewed the material from the class prep assignment for about 5-10 minutes and then distributed worksheets for the students to work on in groups. Students worked in groups of three or four at tables that sat four. Moreover, each table had small whiteboards that students could use to work out solutions together. I circulated around the room, giving hints, checking work, and choosing student volunteers to present solutions. The student volunteer displayed his/her solution to a problem using a document camera at the front of the room and explained his/her steps. During this short presentation, other students could ask questions and make comments. I used this opportunity to highlight whether a particular solution would receive full credit on an exam or if there was some notation or explanation missing. I typically selected problems on which students make common errors. See Section 7.f for the group work assignment that corresponds to the class prep assignment shown in Section 7.b. Visit `https://www.youtube.com/watch?v=pN0HjbnuJ_M` for a short clip of one of my classes, in which I motivated the need for a precise "$\epsilon - \delta$" definition of a limit.

The final 50-minute class meeting of the week served as a "closure" meeting, during which I reviewed topics with which students were still having trouble, I covered worksheet problems that the groups were not able to complete, and I administered a short quiz. Students were able to provide feedback online before this 50-minute class so that I knew the material they would like to review.

At the end of every week, I met with the flipped Calculus development team to discuss what adjustments could be made to the previous week's group work assignments, what in-class strategies did or did not work, and future assignments.

3.e Technology in the Classroom

I have incorporated technology in many courses. As mentioned in Section 3.d, I created class-prep videos for my flipped Calculus I class and I regularly used a document camera for student presentations. Moreover, certain problems in the group work assignments required students to interpret a graph or an animation. Thus I used the computing program *Mathematica* and the related computing website *Wolframalpha.com* to display animations and graphs via the overhead projector during these class meetings. See problem 1(a) of the group work assignment in Section 7.f for an example of such a problem.

When I taught Calculus II, I used iClickers, which greatly helped facilitate engagement and collaboration in the classroom. This polling software allows students to answer multiple choice problems using a remote. Throughout my Calculus II lectures, I would typically display (via the overhead projector) "quick" multiple-choice problems that evaluated student understanding of the concepts that we had just covered. Students would think about the problems individually for a minute or two, then share their ideas with their neighbors, and finally input their answers. I would then display an answer distribution in the form of a bar graph and if most students got the question correct, I would move on without much explanation. If a sizable portion of the class answered the question incorrectly, I would ask for a student volunteer who answered correctly to explain how he/she arrived to the correct answer. See Section 7.g for sample iClicker problems that I used throughout the semester.

3.f Online Teaching

With the mandatory move to remote learning amid the coronavirus pandemic in Spring 2020, I conducted my lectures and office hours through Zoom. For the sake of consistency, I structured my online lectures in a similar manner as in-class lectures, while taking full advantage of Zoom's whiteboard capability on my tablet to produce and record live lectures during the usual class time. The recordings were posted online in case students could not attend the virtual class or wanted to review the lecture later in the semester. Visit `https://youtu.be/c9_M1PIznaI` and `https://youtu.be/KAG1oC9BHCo` to view two of my Linear Algebra lectures, which covered Markov Chains. See Section 7.e for a homework assignment I created including Markov Chain problems. See Section 6.a.1 for student comments regarding the efficacy switching class from in-person to online instruction.

Although the student comments in Section 6.a.1 are generally positive, a common sentiment by other students was that they lacked the motivation to attend virtual classes. Moreover, in retrospect, I believe that my virtual classrooms lacked an important aspect of the college experience—socialization. To address these shortcoming, in the fall of 2020, I collaborated with other section instructors to implement a virtual "flipped" classroom. With this structure, students watch class prep lecture videos before coming to class and then during class meetings, we discuss the videos and engage in group work (which will be facilitated by Zoom's breakout rooms). You can find a class prep video that I created at https://www.youtube.com/watch?v=3_s_MMOWydw.

4 Mentoring and Student Research

During the 2019-2020 academic year, I worked with an independent study student who opted to take a 1-credit add-on to my Advanced Multivariable Calculus course in Fall 2019 and then took a 3-credit independent study with me in Spring 2020 to continue learning about the field of topology. For the 1-credit add-on, the student used tools developed in Advanced Multivariable Calculus, to explore the following topics related to the theory of 2-dimensional surfaces:

- Morse theory on surfaces,
- Gaussian and mean curvature of surfaces, and
- the classification of surfaces (via triangulations and Euler characteristic).

In the 3-credit independent study in Spring 2020, this student built on the knowledge from the previous semester by exploring the following topics related to the theory of manifolds:

- point-set topology,
- basic definitions of smooth and topological manifolds,
- basic constructions of manifolds (products, connected sums, handlebody decompositions)
- the classification of 1-dimensional manifolds (using point-set topology), and
- the fundamental group.

In the 2020-2021 academic year, along with another faculty member, I will work with this student on a Senior Thesis, which will explore knot theory. In particular, topics to be explored include (but are not limited to):

- basics of knots,
- Seifert surfaces and signature,
- slice knots,
- Alexander and Jones polynomials,
- Legendrian knots, and
- Knot contact homology.

5 Student Evaluation Data

5.a University of Massachusetts Amherst

The following data is taken from student evaluations from select classes at the University of Massachusetts Amherst. Each category is rated out of 5. Both my average and the total averages across all classes in the Mathematics Department with similar class sizes during the same semester are provided.

1. What is your overall rating of this instructor's teaching?

Course	My Average	Department Average
Calculus III	4.7	4.0
Linear Algebra	4.8	4.0
Advanced Calculus	4.7	4.0
Complex Variables	4.8	4.0

2. Instructor showed an interest in helping students learn.

Course	My Average	Department Average
Calculus III	4.7	4.4
Linear Algebra	4.8	4.4
Advanced Calculus	4.8	4.4
Complex Variables	4.9	4.4

3. Instructor explained course material clearly.

Course	My Average	Department Average
Calculus III	4.8	4.1
Linear Algebra	4.9	4.1
Advanced Calculus	4.8	4.1
Complex Variables	4.8	4.1

4. Instructor cleared up points of confusion.

Course	My Average	Department Average
Calculus III	4.8	4.1
Linear Algebra	4.9	4.1
Advanced Calculus	4.8	4.1
Complex Variables	4.9	4.1

5. Instructor inspired interest in the subject matter of this course.

Course	My Average	Department Average
Calculus III	4.4	3.9
Linear Algebra	4.5	3.9
Advanced Calculus	4.7	3.9
Complex Variables	4.5	3.9

6. Instructor stimulated student participation.

Course	My Average	Department Average
Calculus III	4.3	3.9
Linear Algebra	4.3	3.9
Advanced Calculus	4.5	3.9
Complex Variables	4.3	3.9

7. Instructor was well-prepared for class

Course	My Average	Department Average
Calculus III	4.9	4.6
Linear Algebra	5.0	4.6
Advanced Calculus	5.0	4.6
Complex Variables	5.0	4.6

5.b University of Virginia

The following data is taken from student evaluations at the University of Virginia. Each category is rated out of 5. Both my averages and the total average across all sections of the same course are provided.

1. Please rate the instructor

Course	My Average	Course Average
Calculus I	4.66	4.13
Applied Calculus I	4.71	4.18
Calculus II	4.64	4.31
Applied Calculus II	4.68	4.15
Calculus III	4.64	4.31

2. Overall, the instructor was an effective teacher.

Course	My Average	Course Average
Calculus I	4.59	4.02
Applied Calculus I	4.59	4.05
Calculus II	4.57	4.18
Applied Calculus II	4.55	4.00
Calculus III	4.57	4.18

3. The instructor showed a scholarly grasp of the material.

Course	My Average	Course Average
Calculus I	4.66	4.37
Applied Calculus I	4.56	4.35
Calculus II	4.57	4.46
Applied Calculus II	4.65	4.40
Calculus III	4.22	3.86

4. The instructor made good use of examples and illustrations.

Course	My Average	Course Average
Calculus I	4.62	4.00
Applied Calculus I	4.35	4.00
Calculus II	4.46	4.15
Applied Calculus II	4.50	3.98
Calculus III	4.31	3.96

5. A positive environment was provided for student problems.

Course	My Average	Course Average
Calculus I	4.50	4.13
Applied Calculus I	4.36	4.13
Calculus II	4.61	4.32
Applied Calculus II	4.48	4.14
Calculus III	4.33	4.13

6. Students' problems were handled well.

Course	My Average	Course Average
Calculus I	4.62	3.96
Applied Calculus I	4.44	4.02
Calculus II	4.46	4.22
Applied Calculus II	4.58	4.00
Calculus III	4.28	3.97

7. Lectures were clear and well organized.

Course	My Average	Course Average
Calculus I	4.61	3.91
Applied Calculus I	4.44	3.96
Calculus II	4.46	4.00
Applied Calculus II	4.45	3.87
Calculus III	4.42	3.85

8. The instructor seemed to be aware of whether the class was following the presentation.

Course	My Average	Course Average
Calculus I	4.09	3.60
Applied Calculus I	3.73	3.58
Calculus II	4.30	3.77
Applied Calculus II	4.08	3.61
Calculus III	3.64	3.58

6 Student Comments

6.a University of Massachusetts Amherst

Calculus III:

- “Professor Simone is the man! He’s a very approachable person and has very clear lectures. He doesn’t waste time, encourages us to learn and is very engaging. He answers his emails fast and does his best to help us learn in the most comfortable environment possible.”
- “His explanations flowed naturally and always kept us aware of the bigger concepts when addressing finer details. I think, too often in other classes do we dive into the details and lose sight of the whole idea behind them.”
- “I like how he builds concepts from the bottom up, often referring to calc I to build calc III concepts. I also like how he will go through the important steps and examples several times.”
- “I liked that he gave us time to complete problems on our own and asked problems...before he would do it on the board.”
- “Professor Simone is clearly passionate about the material. He showed up to class extremely prepared and had a good answer to any problems that were asked.”
- “He was able to help me after my poor performance on the first exam, and I brought my grade up by nearly 30 points. Overall, excellent teacher.”
- “Prof. Simone was really excited about math and it made it easy to want to come to class, which in a fairly dry subject is pretty surprising.”
- “THE BEST professor I have had at UMass so far.”

Linear Algebra:

- “Dr. Simone facilitated active engagement of students. He frequently required us to think and created an environment in which we were allowed to be wrong openly. He was always prepared with examples and I thoroughly enjoyed his teaching style.”
- “I really liked how Prof Simone created an environment where it felt safe to ask any problems whenever you have them. Often, professors instinctively respond with a little contempt in their voice, but Prof Simone was very patient and helpful.”
- “Professor Simone was a great professor, explained things and went over material multiple times if need be. Very accessible and seemed to actually care about his students.”
- “There was a good balance of examples, proofs, and explanation of concepts, which allowed students to learn and understand in multiple ways. I also did like the balance of written homework and online homework.”

- "I want to thank Professor Simone for a great semester. He is by far the best math Professor I have had at UMass, and this is the 5th math class that I have taken, so I've seen a solid sample. Outside of math, Jonathan Simone is one of the best professor's I have had at my time here, period. Other Professors should strive to match his excellence."

- "I think the highlight of the class was being taught by one of the most skilled educators at UMass. Jonathan is in a league of his own in terms of presentation skills, esp when you consider the topic matter is linear algebra. His neatness in writing material on the blackboard is very conducive to taking clean notes. Overall 11/10 experience, would take again."

Advanced Calculus:

- "He speaks deliberately and clearly, and he makes everything explicit. In addition to explaining concepts verbally, he also writes out his reasonings on the board as well, which makes for great, flowing notes. Granted, I'm not the best notetaker, but I treasure my notes from this class. The homework were also great at getting one to understand the material."

- "Your teaching style is excellent. I loved how you motivated the material and gave a clear and meaningful explanation of the ideas you were presenting. Your lectures were almost conversational and it was a great environment to learn in."

- "There should be an option for"always" not "almost always" for the first 7 problems. Dr. Simone fits that role. He is absolutely incredible; by far, one of the best professors I have ever had. He makes himself so accessible to students, gives insightful feedback, and genuinely cares about his students' success. His lectures are also incredibly well organized, intriguing, stay true to the lesson he is trying to get across, or lead you to the problems he is hoping you ask of the material. Dr. Simone uses his time in lecture as if it were money: wisely and with intent. Absolutely brilliant and kind person."

- "I liked how he always did example problems even for easy problems just for our own reference when looking back in the notes a month later and we can refresh ourselves with the material"

- "I really enjoyed the emphasis on the geometric interpretations of the concepts we were learning. This made it much easier to intuitively understand why the theorems and formulas worked like they did."

- "I'm sold on preclass assignments, they should be a part of every class. Very helpful."

- "I love how he managed to relate the material with topology in a very decent manner."

Complex Variables:

- "Overall he is one of the best math professors at UMass. He does a great job of teaching the material and is always helpful in office hours."

- "Jon is absolutely one of my favorite professors. I am always amazed at how he is organized and find the best way for students to learn the material. I have taken several classes of his, and I still use his lecture notes to reference other classes. Before the semester starts, my friend, who took 421 a few semesters ago, scared me for being very challenging. However, Jon made the class very easy to follow and was very helpful during office hours that I never thought this course being that challenging. It is actually one of my favorite courses in college."

- "The material is fascinating and the instructor is very engaging. He presents the material very clearly and develops unfamiliar concepts logically starting from concepts which are already deeply familiar to students. The material is presented so well that these extremely non-trivial results feel inevitable by the end of each unit."

- "You are one of the reasons why I love being a math major."

6.a.1 On Switching to Online Learning Mid-Semester

- "Professor Simone was awesome during the switch to remote learning, he kept consistency with his live lectures and made them feel like they were in the real classroom. The whiteboard section of Zoom was just like the chalk board. Class participation was about as frequent as in face to face instruction."

- "Prof. Simone had perhaps the most effective online lectures out of my four courses. He quickly realized that students were gravitating toward using the chat box as a primary form of communication and paid close attention to it to always address any questions presented."

- "The switch to remote learning for this class was pretty seamless, as if it had been done many times before. Students could still be just as interactive on here as they were in person."

- "You still helped me when I had questions. You were also still engaging and easy to follow during the lectures. Thank you!"

6.b University of Virginia

Calculus I:

- "I really liked Professor Simone and have already recommended him to others. He was very approachable and willing to work with students. He genuinely seemed to care about the success of each student in the class. He presented all of the material in a way that could be understood clearly, and he prepared us well for all homework, quizzes, and tests."

- "Confident teaching style, able to gauge whether students understood information"

- "His office hours were always helpful and he gives off the feeling that he's very devoted to math."

Applied Calculus I:

- "Jonathan Simone is probably the best math teacher I've ever had. Loved him. He actually managed to make math fun, interesting, and even at times funny. I've recommended him to all my friends. He's clear, intelligent, and explains everything extremely well. Give him a raise and a corner office!"

- "My instructor was amazing at helping me pass this course. I went to his office hours close to every week and when I could not make it to his office hours, he made time for me. He was very available and he would explain the material to me in different ways when he saw that I didn't understand."

Calculus II:

- "I've honestly never been a math person, but Prof. Simone is really great - he takes time to talk to students, explain things thoroughly, and is always upbeat...he's probably the best math teacher I've ever had."

- "I've heard plenty of horror stories regarding less-than-illustrious Calc-II classes. This wasn't one of them. Jon did a good job with the material."

Applied Calculus II:

- "I thought Professor Jonathan Simone was excellent. The learning environment was fun. I thought he was always well prepared for class, he taught at a good pace, always made a point to open up space for problems (at the very beginning of class and after covering a segment of material during class), he answered problems adequately, and had a good pulse for how well students were retaining information in general. I think he did a great job explaining the material in class, and was a great help and very approachable outside of class during office hours. It also helped that he was flexible with students. I had a great experience in his class."

- "Very approachable, helpful, and a good instructor"

- "Jon was a great teacher; he recognized the challenging nature of the material and went out of his way with office hours to help us learn as best we could."

Calculus III:

- "Professor Simone is excellent. This is the second course that I have taken with him, and I was thrilled to find out that he was teaching my class again. He is extremely approachable and helpful, and I've always thought that he truly cares about how his students perform in his courses."

- "I went to Jon's office hours every week, sometimes twice a week. He was always so helpful. He was always able to convey the material effectively during his lectures. I really want to have another class with him if possible. Excellent doesn't even begin to describe how freaking awesome he was. On every scale I give him a 10/10. "

7 Sample Teaching Materials

7.a Sample Syllabus

University of Massachusetts Amherst
MATH 425 - Sections 1 and 3
Advanced Calculus Syllabus - Fall 2019

Instructor: Jon Simone
Email: jsimone@umass.edu
Office: LGRT 1332
Office Hours: M: 12-1pm, W: 2-3pm, F: 11:30-12:30pm, and by appointment
Class Website: https://people.umass.edu/jsimone/fall2019math425.html
Textbook: Vector Calculus (6th edition) by Marsden and Tromba

Overview: This course will explore differential, integral, and vector calculus in n-dimensional space. A strong understanding of single variable calculus, multivariable calculus, and linear algebra is expected. Throughout the semester, I may cover topics and applications that are not in the textbook. Thus, it is important for you to regularly attend class.

Throughout the semester, please regularly vist the class website (https://people.umass.edu/jsimone/fall2019math425.html) to keep up-to-date with upcoming assignments and due dates.

Pre-Class Assignments: To get the most out of class, you will be assigned short pre-class assignments to prepare you for the upcoming class material. These assignments will either: (1) review pre-requisite concepts from calculus and linear algebra; (2) review material from a previous lecture; (3) or include short readings from the textbook. These assignments should be completed before coming to class. They are listed (and regularly updated) on the class website (https://people.umass.edu/jsimone/fall2019math425.html).

You are to upload a picture or scanned copy of these assignments to our Moodle site before class. They will be checked for completeness. If you are unable to upload the assignment for some reason, you can hand in a hard copy of the assignment at the beginning of class. Assignments handed in after the beginning of class will not be accepted.

Since it is hard to predict exactly how much material we will cover on a given day, pre-class assignments are subject to change. Be sure to check the schedule regularly to keep up-to-date. To be safe, I recommend beginning to work on a pre-class assignment no earlier than 1pm on the day of the previous lecture.

Homework: There will be written homework assignments due roughly every two weeks. These will be due at 4pm on the days listed in the schedule on the class website (https://people.umass.edu/jsimone/fall2019math425.html). Assignments are to be put in my mailbox in LGRT 1623D. Since I am teaching two sections of this class, be sure to put your assignment in the correct envelope labelled with your section (either 1 or 3).

Late homework will not be accepted. The lowest homework grade will be dropped. Solutions will be posted after the due date on the class website.

These assignments will be graded on logical flow and clarity as well as mathematical correctness. That is to say, please write neatly and concisely. Using a pencil is alway a good idea. You may work on your own or collaborate with others on these assignments. If you choose to collaborate, be sure to write your solutions in your own words—solutions should not be simply copied from collaborators. I will typically post homework at least a week in advance of the due date on the class website. It is a good idea to begin working on these assignments well before the due date. This will give you plenty of time think about the problems, collaborate, and come to office hours if you have problems.

Exams: There will be three in-class exams and a final exam. The dates and topics are as follows.

Exam 1: Friday, September 27
Will cover the differential calculus topics covered between 9/4 and 9/20.

Exam 2: Friday, October 18
Will cover the integration and parametrization topics between 9/23 and 10/11.

Exam 3: Friday, November 15
Will cover vector calculus topics between 10/15 and 11/8.

Final Exam: Check Spire for you section's final exam time and date.
Will be cumulative.

Grading: All of your grades will be posted on Moodle. The course grade is broken down as follows.

Pre-class assignments	5%
Homework:	20%
Exam 1:	15%
Exam 2:	15%
Exam 3:	15%
Final:	30%

Disability Services Accomodations: The University of Massachusetts Amherst is committed to making reasonable, effective and appropriate accommodations to meet the needs of students with disabilities and help create a barrier-free campus. If you have a disability and require accommodations, please register with Disability Services (161 Whitmore Administration building; phone 413-545- 0892) to have an accommodation letter sent to your faculty. Information on services and materials for registering are also available on their website.

7.b Class Prep Assignments

<u>Select Advanced Calculus Pre-Class Problems</u>

I assigned pre-class problems to my Advanced Calculus class that either: (1) reviewed pre-requisite concepts from calculus and linear algebra or (2) reviewed material from previous lectures. Below is a selection of pre-class problems. The full list of problems (and solutions) can be found at https://people.umass.edu/jsimone/PreClassProblems.pdf. For more details, see the syllabus in Section 7.a.

1. Find the linearization L of:
   (a) (Calc I Review) $f(x) = \sin x + 1$ at $x = \pi$.
   (b) (Calc III Review) $g(x, y) = x^2 + y^2 + 1$ at $(x, y) = (1, 1)$

2. (Linear Algebra Review)
   (a) Find the eigenvalues of $A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$.
   (b) Fill in the blank: An $n \times n$ matrix M has an eigenvalue of 0 if and only if $\det M =$ ____.
   (c) True/False : A symmetric $n \times n$ matrix M (i.e. $M^T = M$) has n real eigenvalues.

3. (Review of Previous Lecture)
   (a) Write down the general second derivative test from last lecture.
   (b) Show that $(0, 0, 0)$ is a critical point of $f(x, y, z) = x^2 + y^2 + z^2 + yz$. Use the second derivative test to show that it is a local minimum.
   (c) Show that $(0, 0)$ is a degenerate critical point of $f(x, y) = x^7 - y^4$.

4. (Calculus III Review) Evaluate the following double integrals.
   (a) $\displaystyle\int_0^1 \int_{-1}^1 x^2 y \, dx \, dy$
   (b) $\displaystyle\int_0^1 \int_1^{e^x} (x + y) \, dy \, dx$

5. (Linear Algebra Review): Let $A = \begin{bmatrix} 1 & 2 \\ 1 & -4 \end{bmatrix}$
   (a) Sketch the parallelogram formed by the columns of A.
   (b) Without using geometry, find the area of that parallelogram. (Hint: It is related to the determinant of A.)
   (c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by the matrix A and let $D = \{(x, y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}$. What is the area of $T(D)$ (i.e. the image of D)? (Hint: It is related to the determinant of A and the area of D).

6. Try to wrap an orange in a sheet of paper. What happens to the sheet of paper?

7. (Parametrization Review) Let C be the portion of the unit circle in the first quadrant. It can be parametrized by $\mathbf{c}(t) = (\cos t, \sin t), 0 \leq t \leq \frac{\pi}{2}$.

   (a) What is the orientation/direction of this parametrization (i.e., as t increases, do you move clockwise or counterclockwise)?

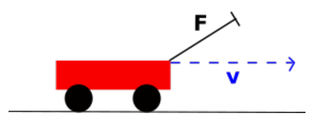
   (b) Show that $\mathbf{c}(t) = (\cos 2t, \sin 2t), 0 \leq t \leq \frac{\pi}{4}$ is also a parametrization of C with the same orientation.

   (c) Reparametrize C so that the orientation is reversed.

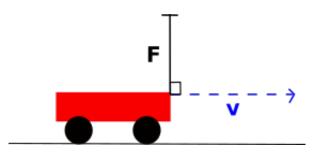
8. (Some Light Physics/Calculus III Review) If $\mathbf{F}$ is a force field in $\mathbf{R}^3$, then an object in $\mathbb{R}^3$ might be moved by $\mathbf{F}$ (e.g. an apple falling from a tree due to a gravitational field). Suppose we want the force field to move an object at rest along a particular path. The **work** done by $\mathbf{F}$ measures how easy it is for $\mathbf{F}$ to move the object along the path.

   If $\mathbf{F}$ is a constant force and the path is a straight line segment given by a vector $\mathbf{v}$, then the work done is given by the dot product $W = \mathbf{F} \cdot \mathbf{v}$.

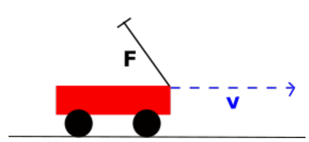
   A classic example is when you pull a wagon. Suppose this wagon is connected to tracks and can only move left or right. The vector $\mathbf{F}$ corresponds to the handle; $\mathbf{F}$ points in the direction in which you are pulling the wagon and $||\mathbf{F}||$ measures how hard you are pulling the wagon. The vector $\mathbf{v}$ is the direction that you want to pull the wagon.



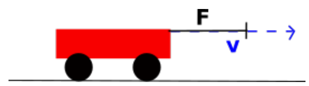
   If the work done is 0, then the force is unable to move the object at all. This occurs when $W = \mathbf{F} \cdot \mathbf{v} = 0$, which geometrically means $\mathbf{F}$ is perpendicular to $\mathbf{v}$. In the wagon example, if you pull the handle straight up, it obviously will not move forward.



   If $W < 0$, then the force moves the object in the "wrong" direction. This occurs when the angle between $\mathbf{F}$ and $\mathbf{v}$ is greater than $90°$.



   If $W > 0$, then the force is able to move the object in the correct direction. The work is maximized when $\mathbf{F}$ and $\mathbf{v}$ point in the same direction. In the wagon example, if you pull the wagon for a minute, you will get the furthest if $\mathbf{F}$ and $\mathbf{v}$ are parallel to each other.



Now for an actual computation: Suppose the handle of the wagon is given by $\mathbf{F} = 2\mathbf{i} + \mathbf{j} = \langle 2, 1\rangle$ and the direction in which we wish to pull it is $\mathbf{v} = \mathbf{i} = \langle 1, 0\rangle$. Compute the work done by $\mathbf{F}$ to move the wagon along $\mathbf{v}$.

9. (Surface Area Review)

   (a) Let S be a surface with parametrization $\mathbf{r}(u, v)$, where $a \le u \le b$ and $c \le v \le d$. Write down the formula for the surface area of S.

   (b) Now suppose S is the graph of a function $z = g(x, y)$. We can parametrize S by $\mathbf{r}(u, v) = (u, v, g(u, v))$. Using this parametrization, compute $||\mathbf{r}_u \times \mathbf{r}_v||$.

   (c) Rewrite the formula of the surface area of S using the computation in part (b).

10. (An art project) Find a long rectangular strip of paper (cut the bottom 2 inches from a regular 8×11 sheet of paper, for example). Draw a straight line down the center of the strip (that is parallel to the long edge). If you glue the ends of the strip together without twisting it, you will have a short cylinder and the line you drew will be on the same side of the strip.

    Instead, twist the strip 180°, as depicted below.

    Now glue/staple the ends together. You are left with what is called a Möbius band. What happened to the line you drew? Is it still on the same side of the strip?

11. (Calculus I Review) Recall, that the differential of a function $y = f(x)$ is given by $dy = f'(x)dx$. More generally, the differential of a function of n variables $z = f(x_1, \dots, x_n)$ is given by $dz = \dfrac{\partial f}{\partial x_1}\, dx_1 + \cdots + \dfrac{\partial f}{\partial x_n}\, dx_n$.

    (a) Let $f(x) = x^2 + x$. Compute the differential of f and use it to estimate the change in f as x changes from 1 to 1.01.

    (b) Let $z = f(x, y) = xy$. Compute the differential of f and use it to estimate the change in f as (x, y) changes from $(1, 1)$ to $(1.01, .08)$.

12. (Review of Last Class) Let $\alpha = 2xz\, dx + 3xy\, dy + 2e^z\, dz$ and $\beta = 3x\, dy - 2\, dz$ be 1-forms on $\mathbb{R}^3$. Compute $\alpha \wedge \beta$.

A Class Prep Assignment for Flipped Calculus I

Visit https://youtu.be/r-Pc20qxEh8 for the class prep video, which I created, associated with these problems.

1. **Question Details** DerivativeasFunctions [3961532]

In the video, you are referred to an example in the textbook. The function in that example not differentiable at some point a. What is a?

- 1
- -1
- 0

Why isn't the function differentiable at a?

- f is not continuous at a
- $\lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} \neq \lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h}$
- f is not defined at a

2. **Question Details** CP4_1 [3778

Each of the following functions fails to be differentiable at $x = 0$.

(A) $f(x) = |x|$

(B) $f(x) = x^{1/3}$

(C) $f(x) = \begin{cases} x\sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$

(D) $f(x) = \frac{|x|}{x}$.

The video as well as the textbook discusses three graphical ways to spot a number at which a function fails to have a derivative. For one of the preceding functions, the non-existence of $f'(0)$ is not explained by the graphical methods described in the video and text. What is the letter of this function?

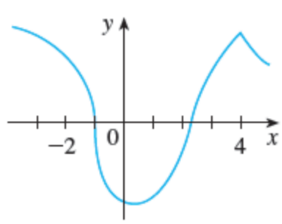
- B
- C
- D
- A

5. **Question Details**

The graph of f is given. State the numbers at which f is not differentiable.

$x =$ ______ (smaller value)

$x =$ ______ (larger value)



7.c Final Exams

Note: The following final exams, which I wrote, do not include the usual formatting, spacing, or coverpages.

UMass Amherst - Complex Variables (2 hours, Open Book)

1. Determine *all* singular points of the following functions (except ∞).
   Classify the isolated singular points and calculate the residue at each one (except ∞).

   (a) $f(z) = \dfrac{(z^6-1)e^{\frac{1}{z}}}{z^4}$

   (b) $f(z) = \dfrac{\text{Log} z}{z-1}$

2. Evaluate the following integrals using any method we discussed in class.

   (a) $\displaystyle\int_C \csc^2 z\, dz$, where C be the contour given by the graph of the function $y = \cos x$ from $x = \frac{\pi}{2}$ to $x = 0$. Express your answer in terms of e.

   (b) $\displaystyle\int_C \frac{z}{(z-2)^2}\, dz$, where C is the positively-oriented circle $|z - i| = 2$.

   (c) $\displaystyle\int_C \frac{z^2-1}{2z^3+2z^2+z}\, dz$, where C is the positively-oriented unit circle.

3. Consider the function $f(z) = \dfrac{\cos^2(\pi z) - 2e^{\sin(\pi z)}}{\sin(\pi z)}$.

   (a) Show that $z = n$ is a singular point of f for all $n \in \mathbb{Z}$ and that

   $$\operatorname*{Res}_{z=n} f(z) = \begin{cases} \frac{1}{\pi} & \text{if } n \text{ is odd} \\ -\frac{1}{\pi} & \text{if } n \text{ is even} \end{cases}$$

   (b) Let $C_{\frac{2k+1}{2}}$ be the positively-oriented circle $|z| = \frac{2k+1}{2}$, where k is a nonnegative integer. Show that

   $$\int_{C_{\frac{2k+1}{2}}} f(z)\, dz = \begin{cases} 2i & \text{if } k \text{ is odd} \\ -2i & \text{if } k \text{ is even} \end{cases}$$

4. Suppose f is analytic at a point z_0. Show that there exists a positive real number R such that if C_r is a circle of radius r centered at z_0 and $r < R$, then $\displaystyle\int_{C_r} f(z)\, dz = 0$.

5. Suppose f is analytic everywhere except at $z_1, z_2 \in \mathbb{C}$. Let C_1 and C_2 be positively-oriented circles of radius r centered at z_1 and z_2, respectively, and let C be a positively-oriented contour enclosing C_1 and C_2. Show that

   $$\left| \int_C f(z)\, dz \right| \leq 2\pi r(M_1 + M_2)$$

   where M_1 and M_2 are the maximum values of $|f(z)|$ on C_1 and C_2, respectively.

6. Is it possible for the power series $\sum_{n=1}^{\infty} n^n z^n$ to be the Maclaurin series expansion of some function f that is analytic at 0?

7. I saw the following calculation on the internet recently:

$$i^2 = \sqrt{-1}\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1 \Rightarrow i^2 = 1 \Rightarrow i = \pm 1$$

There is obviously something wrong with this calculation. What is it?

University of Virginia - Calculus III Final Exam (3 hours)

1. Let L be a line that passes through the points Q and R. Let P be a point not on L. Let $\mathbf{a} = \overrightarrow{QR}$ and $\mathbf{b} = \overrightarrow{QP}$. Show that the distance from P to L is given by $d = \dfrac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$. (Hint: Draw a picture and use geometry)

2. Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 2$ where the tangent plane is parallel to the plane $3x - y = -3z + 1$. Explicitly write down all points.

3. Find and classify all critical points of $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$.

4. Use Lagrange multipliers to find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

5. A projectile if fired with speed $48\sqrt{2}$ ft/s and at an angle of elevation of $45°$ from a position 160 ft above ground level. At what speed does the projectile hit the ground? (Hint: Acceleration is due to gravity, which is -32 ft/s^2. Thus $\mathbf{a}(t) = -32\mathbf{j}$)

6. Use a change of variables to evaluate $\iint_R (x + y)e^{x^2 - y^2}\, dA$, where R is the rectangle enclosed by the lines $x - y = 0, x - y = 2, x + y = 0$, and $x + y = 3$. Be sure to draw both R and the resulting shape in the new coordinate system.

7. Consider the sphere $x^2 + y^2 + z^2 = 16$ and the planes $z = -2, z = 2$.

   (a) Parametrize the curve of intersection C between the sphere and the plane $z = 2$ (write down a vector equation and appropriate bounds). Calculate the arc length of C using the arc length formula.

   (b) Consider the surface S which is made up of the part of the sphere that lies between the two planes. Parametrize S (write down a vector equation and appropriate bounds) and then find the surface area of S using the surface area formula.

   (c) Set up, **but do not evaluate**, the integral needed to find volume of the solid bounded above by the sphere and below by the plane $z = 2$.

8. Let $f(x, y) = 2x + 4y$.

   (a) Set up, **but do not evaluate**, the double integral $\iint_D f(x,y)dA$, where D is the region bounded by $y = 2 - x$, $y = x - 2$, and $x = y^2$.

   (b) At any point (x, y), in which direction does f have the maximum rate of increase?

   (c) Letting the point (x, y) vary in $\mathbb{R}^2$, the answer to part (b) gives a vector field that is defined on all of $\mathbb{R}^2$. Call it $\mathbf{F}$. Let C be any smooth simple closed curve in $\mathbb{R}^2$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. Explain your reasoning.

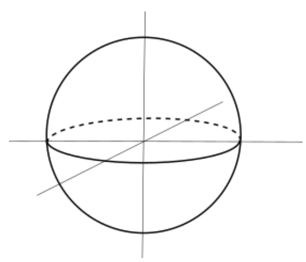
9. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, for the following vector fields and curves. If you use a theorem, cite which one you use.

   (a) $\mathbf{F}(x, y, z) = e^y\mathbf{i} + xe^y\mathbf{j} + 2z\mathbf{k}$ and C is any smooth path from $(0, 1, 1)$ to $(1, 0, 2)$.

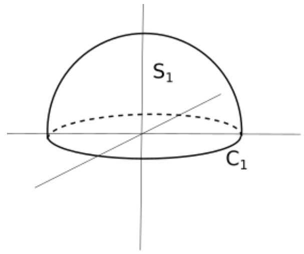
   (b) $\mathbf{F}(x, y) = xe^{-2x}\mathbf{i} + (\frac{1}{4}x^4 + \frac{1}{2}x^2y^2)\mathbf{j}$ and C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first quadrant, oriented clockwise.

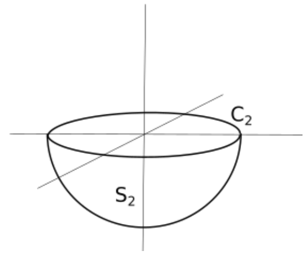
10. Let $\mathbf{F}$ be any vector field satisfying the hypotheses of Stoke's theorem. Let S be the unit sphere centered at the origin in $\mathbb{R}^3$ with the positive/outward orientation $\mathbf{n}$.

    (a) S is shown below. Specify the orientation on S by drawing some normal vectors to S in the correct direction.



    (b) Consider the northern and southern hemispheres of S. Call them S_1 and S_2, as shown below. These hemispheres have boundaries, call them C_1 and C_2, respectively. Notice that C_1=C_2 is simply the unit circle in the xy-plane. Redraw the normal vectors from part (a) on S_1 and S_2 below. The orientations on S_1 and S_2 induce orientations on C_1 and C_2. Using the right hand rule (with your thumb pointed in the direction of the unit normal vector field and the curling of your fingers indicating the orientation on the boundary), draw the induced orientations on C_1 and C_2. Are these orientations the same or different?





(c) Orienting C_1 and C_2 as in part (b), how are $\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ and $\displaystyle\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ related?

(d) Use part (c) and Stoke's theorem to show that $\displaystyle\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S} = 0$.

11. Let $\mathbf{F}(x, y, z) = \langle e^x \sin y, e^x \cos y, yz^2 \rangle$. Find the flux of $\mathbf{F}$ across the surface of the box bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$, and $z = 2$.

12. Suppose a surface S is given by the graph of $z = g(x, y)$ with domain D and let $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$. Parametrize S and show that

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \Big(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\Big) dA$$

13. True/False. Circle ONE answer. You do not have to show your work.

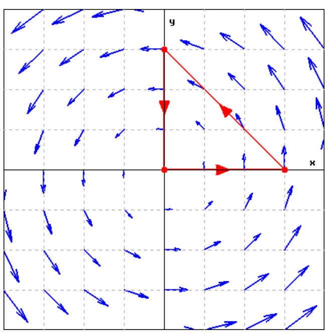
(a) If $\mathbf{F}$ is a vector field defined on $\mathbb{R}^3$, has component functions with continuous partial derivatives, and curl$\mathbf{F}$=0, then $\mathbf{F}$ is conservative.

TRUE **FALSE**

(b) $\displaystyle\int_{-C} f(x, y) ds = -\int_C f(x, y) ds$

TRUE **FALSE**

(c) In the figure below, the work done by the vector field to move a particle along the given bold curve is positive.

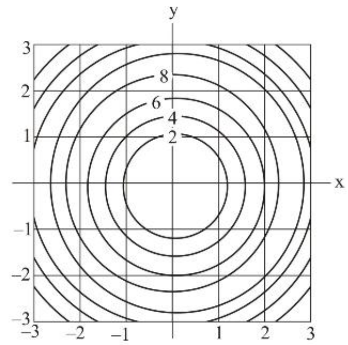


TRUE **FALSE**

(d) Two planes perpendicular to a third are parallel.

TRUE **FALSE**

(e) Consider the contour map for a function f below. The directional derivative of f at $(2, 0)$ in the direction of $\mathbf{j}$ is positive.



TRUE **FALSE**

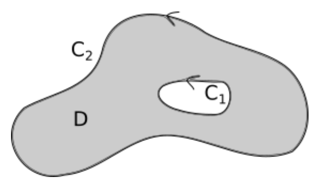
(f) If a function of two variables f is defined on a closed, bounded region D in the xy-plane, then f attains an absolute maximum and absolute minimum on D.

TRUE **FALSE**

(g) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$ for any two nonzero vectors $\mathbf{a}, \mathbf{b}$.

TRUE **FALSE**

(h) Consider the region D depicted below. Suppose $F = P\mathbf{i}+Q\mathbf{j}$ is a vector field whose components have continuous partial derivatives in an open region containing D. Then, by Green's Theorem, $\displaystyle\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_{C_1} \mathbf{F} \cdot d\mathbf{r} - \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$



TRUE **FALSE**

(i) Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives. Then div($\nabla\mathbf{F}$)=0.

TRUE **FALSE**

(j) Any vector field of the form $\mathbf{F}(x, y, z) = f(y, z)\mathbf{i} + g(x, z)\mathbf{j} + h(x, y)\mathbf{k}$ is incompressible.

TRUE **FALSE**

John Jay College - Finite Mathematics Exam 2 (75 minutes)

1. A jar contains 3 red marbles, 4 yellow marbles and 2 white marbles.

   (a) If you select 1 marble at random from the jar, what is the probability that

      i. the marble is white?

      ii. the marble is red or yellow?

(b) If you select 2 marbles at random, without replacement, what is the probability that

   i. both marbles are white?
   ii. one is red and one is white?
   iii. the second marble is red if you know that the first marble is white?

2. Let $P(A) = .3, P(B) = .5$, and $P(A \cap B) = .1$ Find:

   (a) $P(A|B)$
   (b) $P(B|A)$
   (c) $P(\overline{A})$
   (d) $P(A \cup B)$

3. You flip a coin three times. If you flip 3 heads, you win \$2. If you flip 2 heads, you lose \$2. If you flip 1 head, you win \$1. If you flip no heads, you don?t win or lose anything. What are your expected winnings for this game? Is it fair? If not, how much should you lose when you flip 2 heads to make it fair?

4. A coffee company sells two kinds of beans – Ethiopian Harar and Sumatra. 30% of their stock is Ethiopian Harar and 70% is Sumatra. 1% of the harar crop yields bad beans and 2% of the Sumatra crop yields bad beans. You are given a pound of coffee from this company and are told that beans are bad. What is the probability that the beans are Ethiopian Harar?

5. The probability that I eat a peanut butter jelly sandwich for lunch is .25. The probability that Morgan Freeman eats a peanut butter and jelly sandwich for lunch is .1 (I thoroughly researched this fact). Assuming independence, what is the probability that:

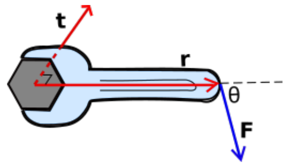
   (a) Morgan Freeman and I will both have peanut butter and jelly for lunch today?
   (b) Neither of us will have peanut butter and jelly for lunch today?

6. A true-false exam has 30 problems. A student randomly guesses on every question. What is the probability that he gets:

   (a) exactly 10 problems correct?
   (b) at least 28 problems correct? (Hint: The binomial probability formula would work nicely here)

7.d Calculus III fourth-hour discussion worksheet

Torque measures the tendency of a force to rotate a rigid body about an axis. For example, if you tighten a bolt by applying force to a wrench, the torque measures the tendency of the force to rotate the bolt about its center.



More precisely, if $\mathbf{F}$ is a force acting on a rigid body at a point given by a position vector $\mathbf{r}$, then the torque $\boldsymbol{\tau}$ (relative to the origin) is defined to be the cross product:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

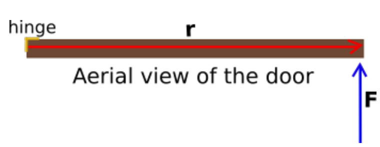
(Recall that the cross product is *not* commutative, so the order of the vectors matters.) Notice that this mathematical definition of torque matches the physical definition above. For instance, if the torque vector is zero, then we know that the vectors $\mathbf{r}$ and $\mathbf{F}$ are parallel. This means that you are either pushing the wrench towards the bolt or pulling it away from the bolt and so you will not turn the bolt. Thus the tendency of this force to rotate the bolt about its center is 0.

Since $\boldsymbol{\tau}$ is a cross product, we have that:

- The direction of $\boldsymbol{\tau}$ indicates the axis around which the body rotates. Using the right hand rule, you can determine the direction of $\boldsymbol{\tau}$ (but first, you need to situate the vectors $\mathbf{r}$ and $\mathbf{F}$ so that they have the same initial point).
- The magnitude of $\boldsymbol{\tau}$ is $\tau = |\boldsymbol{\tau}| = |\mathbf{r}||\mathbf{F}|\sin\theta$, where θ is the angle between the position and force vectors. The magnitude is measured in *newton meters* (N·m). (Note: We do not use joules to measure torque, even though a joule is also defined to be N·m. The word joule is reserved for measuring energy and work. Torque is an entirely different concept. So to avoid misunderstandings, torque is measured in newton meters)
- If $\mathbf{n}$ is the unit vector in the direction of $\boldsymbol{\tau}$, then $\boldsymbol{\tau} = |\boldsymbol{\tau}|\mathbf{n}$.

Example 1: A bolt is tightened by applying a 40N force to a .25m wrench. If the angle between the position and force vectors is 45°, find the magnitude of the torque about the center of the bolt.

Example 2: You rush through a (frictionless) door that is 1m wide by pushing on the edge opposite the edge with the hinges. If you apply a force of 50N perpendicularly to the door, what is the magnitude of the torque about the hinges? Which way does the torque vector point, in relation to the door?



7.e Written Homework Assignments

University of Massachusetts Amherst - Linear Algebra
Math 235-05 Written HW 3
Due Friday, April 17th by 5pm

Directions: Show all of your work. Once complete take **clear** pictures of your homework (preferably with a scanner app), save it as a **single pdf** file, and upload it to Moodle before the deadline. Save your file using the following format: **Lastname_Firstname_HW5**.

1. Let A, B, and C be $n \times n$ matrices. Show that if A is similar to B and B is similar to C, then A is similar to C.

2. (a) Let A be a square matrix and let λ be an eigenvalue with corresponding eigenvector $\mathbf{v}$. Show that $\mathbf{v}$ is an eigenvector of A^2. What is the corresponding eigenvalue?

   (b) Let $B = \begin{bmatrix} -3 & 1 & 182 & -1 & 3 \\ 0 & \sqrt{2} & -3 & 7 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$. Use part (a) to find the eigenvalues of B^2. Explain your reasoning.

   (c) B is diagonalizable. Explain why.

   (d) Since B is diagonalizable, so is B^2 (we showed this in class). Without performing calculations, what are the dimensions of the eigenspaces corresponding the eigenvalues of B^2 that you found in part (b)? Explain your reasoning.

3. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

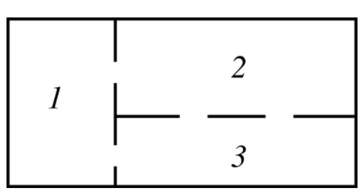
   (a) Explain why P is a stochastic matrix.

   (b) Find the eigenvalues of P.

   (c) Find the dimension of the eigenspace corresponding to the eigenvalue 1.

   (d) Use part (c) to explain why P does not have a unique steady-state vector and find two steady-state vectors for P.

4. A scientist places a mouse in a box with 3 compartments, as shown below.



Notice that there are two doors between compartments 2 and 3, there is one door between compartments 1 and 2, and there is one door between compartments 1 and 3. The mouse has been trained to select a door at random and move through it whenever a bell is rung.

When the mouse is in compartment 1, there is a 1/2 probability that it will move to compartment 2, a 1/2 probability that it will move to compartment 3, and a zero probability that the it will stay in compartment 1. When the mouse is in compartment 2, there is a 1/3 probability that it will move to compartment 1, a 2/3 probability that it will move to compartment 3, and a zero probability that it will stay in compartment 2. Similarly, when the mouse is in compartment 3, there is a 1/3 probability that it will move to compartment 1, a 2/3 probability that it will move to compartment 2, and a zero probability that it will stay in compartment 3. This data can be put into the following stochastic matrix

$$P = \begin{bmatrix} 0 & 1/3 & 1/3 \\ 1/2 & 0 & 2/3 \\ 1/2 & 2/3 & 0 \end{bmatrix}$$

(a) Show that $\mathbf{q} = \begin{bmatrix} 1/4 \\ 3/8 \\ 3/8 \end{bmatrix}$ is a steady-state vector for P.

(b) Show that $\mathbf{q} = \begin{bmatrix} 1/4 \\ 3/8 \\ 3/8 \end{bmatrix}$ is the only steady-state vector for P by computing the dimension of the eigenspace corresponding to the eigenvalue 1.

(c) Suppose the mouse is placed in compartment 1. Then at the start of the experiment, the initial probability vector is $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. This vector tells us that the probability that the mouse is in compartment 1 is 1 and the probability that the mouse is in compartment 2 or 3 is 0. Consider the Markov chain $\mathbf{x}_k = P^k\mathbf{x}_0$. After the kth ring of the bell, the ith entry of $\mathbf{x}_k$ is the probability that the mouse is in the ith compartment (where $i = 1, 2,$or 3). Compute $\mathbf{x}_1, \mathbf{x}_2$, and $\mathbf{x}_3$. After the 3rd ring of the bell, what is the probability that the mouse is in the 2nd compartment?

(d) Without computation, find $\lim_{k\to\infty} \mathbf{x}_k$, where $\mathbf{x}_k$ is the Markov chain from part (c). Explain your reasoning.

(e) If the mouse starts in compartment 2 or 3 instead, what does the Markov chain converge to?

(f) Assuming that the bell is rung at regular intervals, what does the steady-state vector tell you about how much time the mouse spends in each compartment, in the long run?

University of Virginia - Applied Calculus II

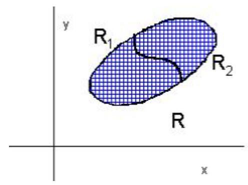
Math 1220-003 Written HW 3

Due Friday, October 21st at 12:00pm

Directions: Be thorough and neat in your explanations and computations. Part of your grade will be based on the presentation of your solutions.

1. Recall the following property of integrals: If f is integrable on $[a, b]$ and $a < c < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$. A similar property is true for double integrals: if $R = R_1 \cup R_2$ (as in the figure below), then

$$\iint_R f(x,y)dA = \iint_{R_1} f(x,y)dA + \iint_{R_2} f(x,y)dA$$



Equipped with this formula, you can integrate over more complicated regions (i.e. regions that are more complicated than the regions we have seen in class).

Let f be a function defined on the region R bounded by $y = x^2, x - y = 3, x = 0$, and $y = 4$.

   (a) Sketch the region R in the $xy-$plane. Break it up into two subregions, R_1 and R_2 that are easy to integrate over. Label these regions.

   (b) Set up an expression for $\iint_R f(x,y)dA$, using the formula above and using the bounds of the regions you found in part (a).

2. Let x be a continuous random variable whose probability density function is $f(x) = \frac{1}{x^2}$ on the interval $[1, \infty)$.

   (a) Find the value of c such that $P(x \geq c) = \frac{1}{10}$. Be sure to use appropriate improper integral notation.

   (b) What is the probability that $x \leq c$?

3. Let f be a probability density function of a random variable X on $[a, b]$. If $a \leq c \leq d \leq b$, then geometrically explain why $P(X \leq c) \leq P(X \leq d)$. What could you say about $P(c \leq x \leq d)$ if $P(X \leq c) = P(X \leq d)$? Explain.

4. Let X be a uniformly distributed random variable on the interval $[a, b]$

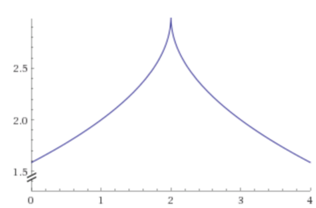
   (a) Prove that $E(X) = \dfrac{a+b}{2}$

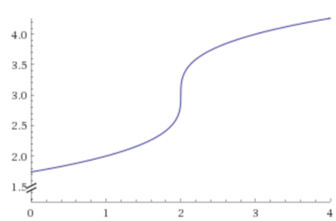
   (b) Prove that $Var(X) = \frac{1}{12}(b-a)^2$

7.f A Group Work Assignment for Flipped Calculus I

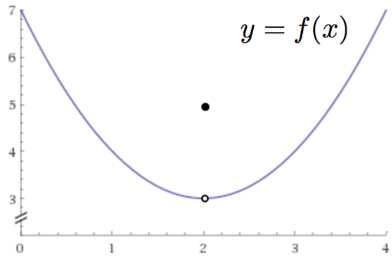
The following is a group work assignment I used in the flipped classroom setting. To save space, I removed extra space provided for student work between problems.

MATH 1310 Derivatives (2.7, 2.8)
Group Members:

1. A particle travels along the x-axis such that its position along the axis at time t is given by $p(t) = 4t - t^2$. Assume p is measured in centimeters (cm) and t in seconds (sec). Thus, e.g., at time $t = 3$ sec, the particle is $p(2) = 3$ cm to the right of the origin.
   (a) An animation of the particle's motion over the time interval $0 < t < 6$ will be shown on the front-of-classroom screen. Based on the animation, for what values of t is the derivative $p'(t)$:
      i. positive?
      ii. negative?
      iii. zero?
   (b) Choose a point in your interval from part (a)(ii). Verify your observation by computing the derivative of $p(t)$ at this point.
2. Does $f'(2)$ exist for the following functions? If yes, what is it? If no, why not?
   (a)
   

   (b)
   

   (c)



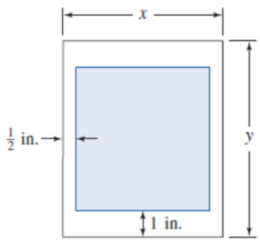
Fill in the blanks: $f(2) =$ ____

$\lim_{h\to 0} (f(2+h) - f(2)) =$ ____.

So what can you say about $f'(2) = \lim_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$?

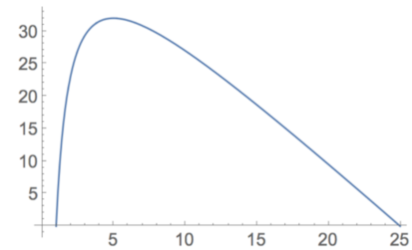
3. **True/False**: If $f(x)$ is continuous at a, then $f'(a)$ exists. Justify your answer.

4. This problem previews a major topic in Calculus: building mathematical models and using calculus to analyze them. A book designer decided that the pages of a book should have 1-in. margins at the top and bottom and 1/2 in. margins on the sides. She further stipulated that each page should have an area of 50 in^2.



The labeling in the figure above and the restriction that $xy = 50$ allows us to express the area f of the printed region (in square inches) as a function of page width x (in inches):

$$f(x) = 52 - 2x - \frac{50}{x}, \quad 1 < x < 25.$$

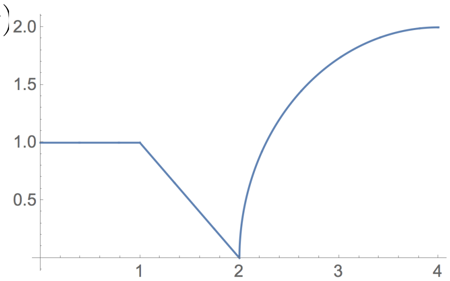


(a) Using the plot of f provided, above right, what value of x appears to maximize the area of the printed region?

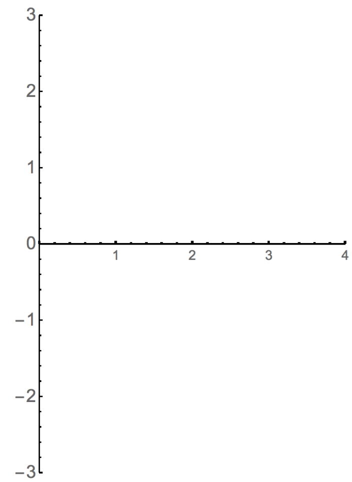
(b) Letting x_0 be the number you identified in (a). What's true of $f'(x_0)$?

(c) Find $f'(x)$ using the definition of derivative. Then use f' and the observation you made in part (b) to confirm your guess in part (a) is correct.

5. Consider the graph of $f(x)$

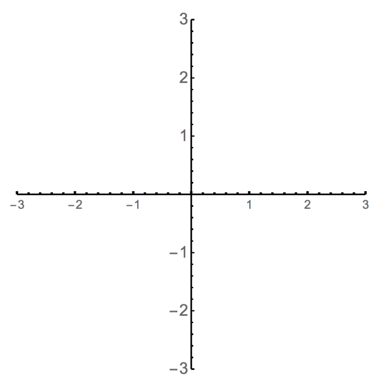


Sketch the graph of $f'(x)$
over the interval $0 < x < 4$



6. (a) Sketch the graph of a continuous function f such that

$$f(0) = 0 \text{ and } f'(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$



(b) What's a formula for the function whose graph you just drew?

(c) What appears to be true of $f'(0)$?

(d) Prove your answer to (c) is correct.

7. Use the definition of derivative to show that if $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x^2}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, then $f'(0)$ exists.

7.g Sample Calculus II iClicker Questions

Question: What method of integration should we use to evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$?

A) u-substitution
B) integration by parts
C) trig sub
D) partial fractions

Question: Which test should we use for $\sum_{n=1}^{\infty} \frac{n}{n^3-1}$?

A) divergence
B) comparison
C) limit comparison
D) integral
E) ratio

Question: Consider $\int_1^{\infty} \frac{x^2+1}{x(x^2-1)} dx$. Let's use the comparison test to determine whether this converges or diverges. What should we compare $\frac{x^2+1}{x(x^2-1)}$ to?

A) $\frac{1}{x}$
B) $\frac{1}{x^2}$
C) $\frac{1}{x^3}$
D) $\frac{1}{x}$

Question: Does $\int_1^{\infty} \frac{x^2+1}{x(x^2-1)} dx$ converge or diverge?

A) converge
B) diverge

Question: Evaluate $\int_0^3 \frac{1}{x-1} dx$.

A) 0
B) $\ln 3$
C) Hold on, buddy.... this is an improper integral. It requires more thought.

Question: Why is $\int_0^3 \frac{1}{x-1} dx$ improper?

A) There is a discontinuity at 0
B) There is a discontinuity at 1
C) There is a discontinuity at 2
D) It slurps its soup

Question: How do we evaluate $\int_0^3 \frac{1}{x-1} dx$?

A) $\lim_{t\to 1} \int_t^3 \frac{1}{x-1} dx$
B) $\lim_{t\to 1} \int_0^t \frac{1}{x-1} dx + \lim_{t\to 1} \int_t^3 \frac{1}{x-1} dx$
C) $\lim_{t\to 1^+} \int_0^t \frac{1}{x-1} dx + \lim_{t\to 1^-} \int_t^3 \frac{1}{x-1} dx$
D) $\lim_{t\to 1^-} \int_0^t \frac{1}{x-1} dx + \lim_{t\to 1^+} \int_t^3 \frac{1}{x-1} dx$

Question: What is the value of $\sum_{n=0}^{\infty} \frac{1}{n!}$?

A) e
B) 1
C) -1
D) 0

Question: Which test should we use for $\sum_{n=1}^{\infty} \frac{1-2^n}{3^n}$?

A) alternating series
B) integral
C) divergence
D) We don't need a test. This is a sum of two geometric series!

Question: What does $\sum_{n=1}^{\infty} \frac{1-2^n}{3^n}$ converge to?

A) $\frac{3}{2}$
B) $-\frac{3}{2}$
C) 2
D) It is divergent

8 Department Service

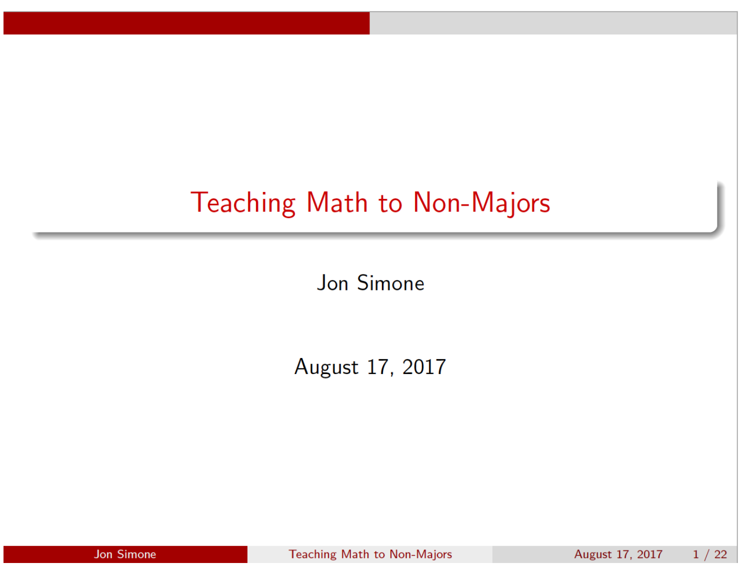
While at the University of Virginia, I volunteered for to share my experiences and knowledge about teaching with new graduate instructors.

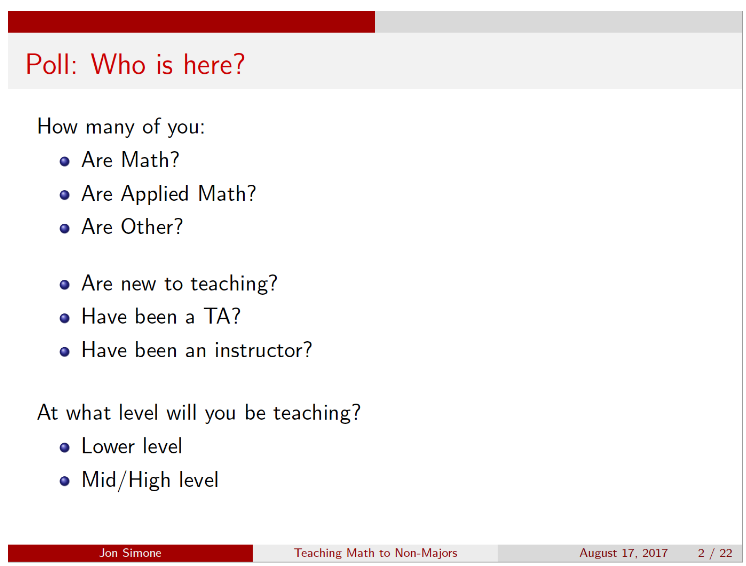
- *Teaching as a Graduate Student Workshop Facilitator*: Every August, the Center for Teaching Excellence at the University of Virginia runs a three-day long teaching workshop for incoming graduate students who will be teaching in the fall. In August 2017, the center invited me to run a workshop titled *Teaching Math to Non-Majors*. During this workshop, I discussed teaching techniques and what could be expected when working with students who are not mathematics majors. Below you can find the slides I used during this workshop.

- *Math Department Teaching Seminar Presentation*: First-year graduate students in the Mathematics Department participate in a teaching seminar in their fall semester. In the fall of 2017, I was invited by the professor who runs the seminar to give a presentation. I spoke about best practices, described the active-learning techniques I employ in my classes, and described how the flipped Calculus I course is run.

- *Math Teaching Mentor*: In this capacity, I served as a peer resource for new graduate student instructors in the Mathematics Department, providing advice, support, and feedback based on class observations.

Teaching Math to Non-Majors Workshop Slides





Your responsibilities as a TA

- Definite responsibilities
  - Coordinating with the instructor or professor
  - Lead fourth-hour discussion
  - Hold office hours
  - Answering student emails
- Possible responsibilities
  - Answering homework questions during fourth-hour
  - Facilitating group work
  - Lecturing
  - Creating quizzes or worksheets
  - Grading
  - Holding review sessions

Tip: Do the work that is assigned to your students

Your responsibilities as an instructor (Math Dept.)

- Structure class and fourth-hour (if you have one)
- Write lectures, quizzes, and written homework (if you want them)
- Grade quizzes and written homework (if you have them)
- Supplement required online homework
- Help develop coordinated exams
- Grading parties for exams
- Hold office hours
- Respond to student emails
- Assign final grades
- Possibly manage a TA
- Anything else you want to add

Challenges especially relevant to teaching math

What are some challenges you have experienced or are worried about?

- Negative student attitudes toward math
- Lack of student confidence
- Under/over prepared students
- Breaking bad habits developed in high school
- Determining a student's conceptual gap
- Avoiding getting too caught up in the details and missing the big picture

Dealing with these challenges

So, how do we deal with these challenges?

- Well... It depends on the student
- Be upbeat and enthusiastic about math
- Keep students engaged. How?
  - Ask questions, group work, use technology
- Encourage students to attend office hours.

Office hours (can be awesome!)

Here are some ways that office hours can benefit you, whether you're an instructor or a TA:

- Get better acquainted with your students
- Gain more insight into a particular student's misunderstanding
- Uncover a topic that most of the class is struggling with
- Help students ask better questions
- Encourage students to work together and teach one another

Office hours may very well be a game changer for certain students

Reaping these benefits requires students to attend office hours, so the next logical thing to consider is...

Getting students to office hours

What factors might encourage students to attend your office hours?

- Your "approachability factor"
- Time and Location
- Reminders
- Structure of office hours

During office hours: Students

Students usually come to office hours to get . . .

- Help with specific questions
- Help with specific concepts
- Help studying for an upcoming exam or quiz
- Help understanding their mistakes on an exam or quiz
- Answers to personal questions relating to their class performance
- Answers to personal questions relating to class logistics

Sometimes a student will have a grading complaint. How should you deal with this?

During office hours:

- What is an example of what **not** to do?
- Ask questions that lead the students through the problem...
  - "Where do you think we should start?"
  - "Why did you pick that method?"
  - "Explain each step in the problem to me."
- Ask students exactly what they don't understand about the topic they are confused about
- Refer back to material they have from class or discussion
- Send students to the board to work out problems
- Stay upbeat and positive
- Start and end on time

The fourth-hour discussion

Factors to consider when deciding on activities for your fourth-hour discussion:

- What does the instructor want (if you are the TA)?
- What do I want the students to gain?
- How interactive do I want discussion to be?
- How will discussion complement material covered in class?

Some common fourth-hour discussion activities

- Question and answer session
- Group work
- Give quizzes
- Review exams

Tip: Make activities as interactive as possible to keep students engaged.

Q&A

- When a question is asked, open it up to the class
- For homework questions, give hints

Tip: Come prepared with relevant problems, just in case no one has questions.

Group work ideas

- Group worksheet
  - 3-4 difficult questions broken into parts
  - TA circulates and helps each group, addressing things on the board if a lot of students are stuck at a certain step
  - Questions prepare students for quiz at the end of discussion
  - Solutions posted online after discussion
- Guided group work.
  - 2-3 difficult problems, broken into individual steps
  - TA allows n minutes per stage, then discusses at the board
  - Students don't fall behind, stay on task, TA doesn't need to rush around

Other ideas

- Preview upcoming material
  - Review of relevant prerequisite material
  - Introduce the next topic and give some basic examples
- Peer Instruction
  - Ask a conceptual question (usually multiple choice)
  - Poll students to see what they think the answer is
  - Have students form groups and discuss their answers
  - Poll students again to see if consensus is reached
  - Good way to get students on board with conceptual groundwork

Teaching flipped Calculus

- Students complete short reading/video assignments before class
- In class, students work on problems in groups
- Instructor circulates, answering questions that arise
- Students present solutions

Special considerations for TAs

- Students may perceive you as an ally against the instructor
- It may be harder for you to keep the students' attention
  - Discussion sections run at 8am/5pm
  - Students may not see you as an authority figure
- Use the fact that you won't have any part in determining exam grades (in most cases), writing exams, or final grades
- Keep the instructor informed about what is working and what isn't
- Be consistent on techniques and terminology developed in class

It's About The Students

- Be aware of how students are responding
  - If students look bored, wake them up
  - If students look confused, start over
  - Students will rarely admit they don't understand
- Collect feedback (and act on it) via short anonymous surveys
- Don't be afraid to change policies or methods mid-semester
  - Make sure to put a blanket statement on your syllabus that says you may change policies mid-semester and that students are responsible for any announcements made in class and emails sent to their UVa email

Finding balance

You are also a student and that is your first priority! Remember:

- Teaching will take as much time as you let it.
- Students will take advantage every time you let them.

It's okay to say no!

Sometimes it's a good thing.

How much time do you want teaching to take up? How will you ensure that you fulfill your duties as a TA or instructor without exceeding your time limit?

Further resources

- Center for Teaching Excellence (CTE): *cte.virginia.edu*
  - Teaching consultations
  - Lecture videotaping (scary, but useful)
  - Workshops (like this one)
- Tomorrow's Professor Today program
  - Two-year program through CTE
  - Seminars on teaching and professional development
  - Applications typically due in May
- Math Department Teaching Seminar (Math 7000)
- Applied Math Department Teaching Seminar (APMA 6548-002)
- Math Department Graduate Teaching Assistant Mentors
- MAA meetings

Questions?

Thank you!

Feel free to contact me!

Jon Simone
js3fv@virginia.edu

9 Outreach and Diversity

The stigmatization of mathematics, especially among underrepresented students, begins at an early age. It is common for students to freely vocalize their difficulties with mathematics with a sense of pride. This attitude is difficult to change once a student reaches high school or college. Thus, this outlook must be curtailed early in a student's education. To help address this at the University of Virginia, I participated in outreach programs aimed at nurturing the mathematical curiosity of young minds.

- *UVa Math Ambassador*: As a UVa Math Ambassador, I visited local elementary schools and facilitated fun mathematics activities for 5th and 6th grade classrooms. These activities explored high-level mathematics topics not covered in the standard curriculum and applied them to fun, interactive problems. For example, one activity introduced the concept of modular arithmetic and applied it to cryptography. In Section 9.a below, you will find an activity that I developed, which explores the basics of topology and Euler characteristic. As a second part to this activity, students would make origami polyhedra and calculate their Euler characteristic. See also `http://indorgs.virginia.edu/MathAmbassadors/` for more details about this program.
- *UVa Math Circle TA*: The UVa Math Circle brings local 4th and 5th grade students together to the UVa campus for two hours a week to learn about high-level mathematics topics. In one session, the students learned about the area minimizing properties of soap bubbles by dipping polyhedra made from plastic vertices and edges into a soap mixture and then observing the result. These meetings are run by a professor in the Mathematics Department and I served as a TA. In this capacity, I worked with students as they worked through these difficult mathematics problems.

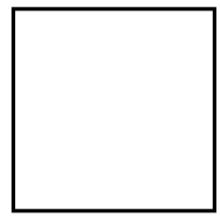
In addition to working with young students, I have also worked with many underrepresented undergraduate students. At New York University and Barnard College, I taught six-week long intensive summer workshop classes for incoming first-year students in the Opportunity Programs. The purpose of these workshops was to prepare students for their first semester of college-level mathematics. My goal for these workshops was to teach my students the mechanics of the course—that is, the necessary computation tools needed to succeed in their fall mathematics courses. For example, when teaching Calculus I at NYU, I would focus on refining my students' algebra and trigonometry skills while having them work through calculus computations such as finding the extreme values and points of inflection of a function. With the computational tools and mechanics under their belts, my students could focus more time on learning the abstract concepts during the fall semester.

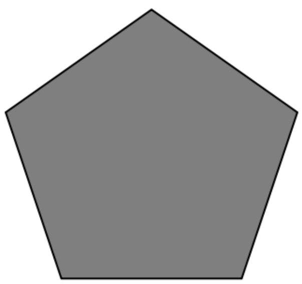
9.a Outreach Activity

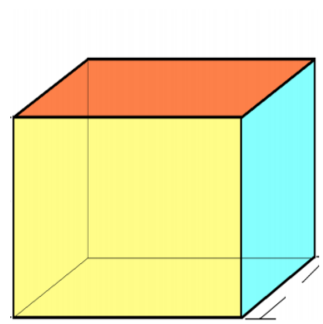
Euler Characteristic

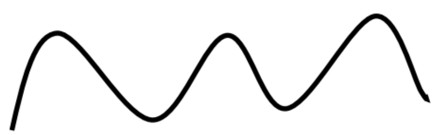
In topology, a branch of mathematics, we study shapes that are not rigid. We imagine that all shapes are made of rubber, so two shapes are the same type if one can be stretched or molded into the other, without breaking it.

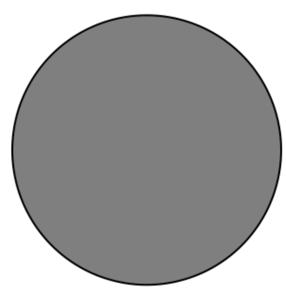
Here are some example of shapes. Which of them do you think are the same type, topologically? Connect the same types of shapes with lines.











In 1752, the mathematician Leonard Euler discovered a simple formula that could tell shapes apart. Today we call this formula the **Euler characteristic**. This is what, in mathematics, we call an **invariant**.

If two shapes have **different** Euler Characteristics, then they are **different**.
If two shapes are the **same**, then they have the **same** Euler characteristic.
Beware: If two shapes have the **same** Euler characteristic, they do **may or may not** be the same.

Euler Characteristic in 2-dimensions

All you need to compute the Euler characteristic is the number of vertices (V) and the number of edges (E). Then the Euler characteristic is given by V-E. It's that simple!

Example 1.1

V = number of vertices (dots) = 2

E = number of edges = 1

V – E = 2 -1 = 1. Thus, the Euler characteristic is 1

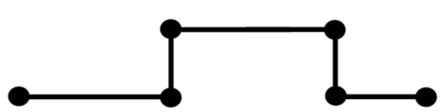
Example 1.2

V = _______

E = _______

V – E = _______

Example 1.2



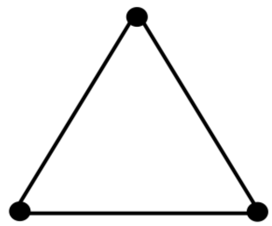
V = _______

E = _______

V – E = _______

Notice that all of these lines have the same Euler characteristic! This makes sense, since they are all the same type of shape.

Example 1.4

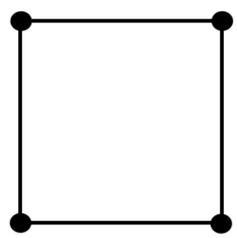


V = ______

E = ______

V – E = ______

Example 1.5

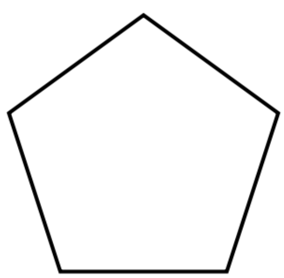


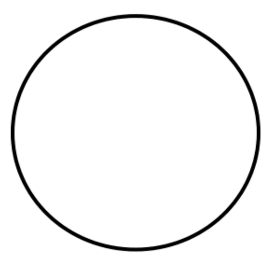
V = ______

E = ______

V – E = ______

Without calculating V or E, what is the Euler characteristic of the following shapes?



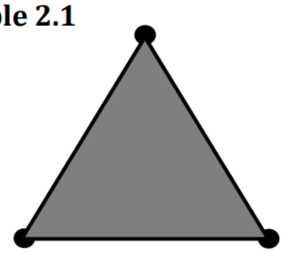


Since all the shapes on this page have Euler characteristic 0, they are **different** than the shapes on the first page.

Euler Characteristic in 3-dimensions

To use this formula in 3 dimensions, you need to know one extra piece of information: the number of faces of the shape. Count the number of vertices (V), the number of edges (E), and the number of faces (F). Then the Euler characteristic is given by V – E + F. Let's try some examples.

Example 2.1



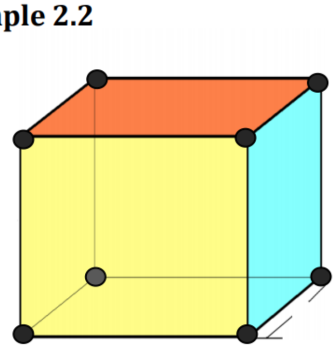
V = _______

E = _______

F = _______

V – E + F = _______

Example 2.2



V = _______

E = _______

F = _______

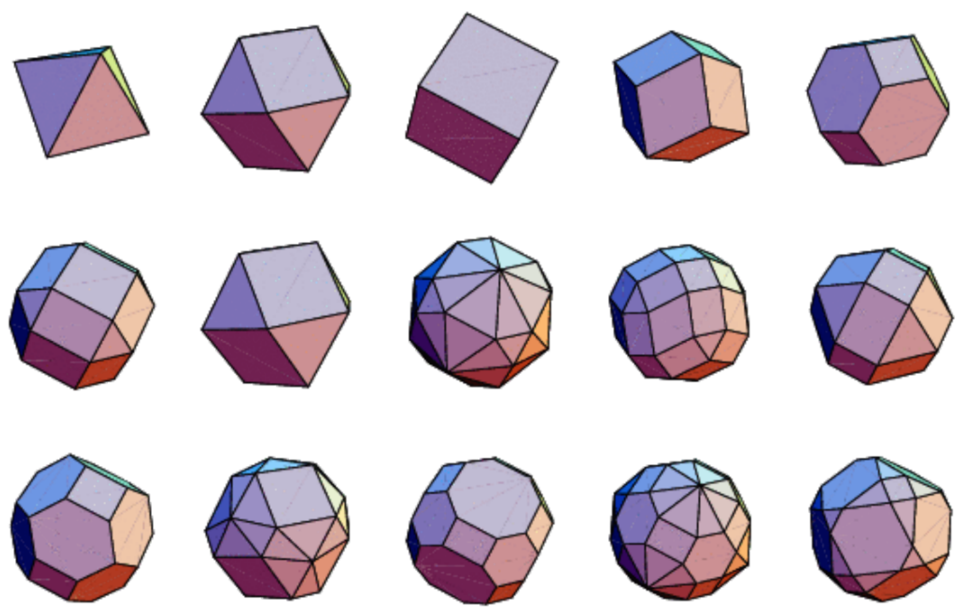
V – E + F = _______

Based on their Euler characteristics, are these shapes the same?

Notice, the Euler characteristic of example 2.1 is the same as the Euler characteristic of the lines in examples 1.1, 1.2, 1.3. This does **not** mean that the triangle and those lines are the same. Obviously they are different types of shapes.

It is usually easy to visualize and draw 2-dimensional and 3-dimensional shapes and so its usually easy to tell which shapes are the same and which shapes are different, without computing the Euler characteristic. Many mathematicians, however, are interested in shapes that are **4-dimensional, 5-dimensional, 6-dimensional, etc.** These are impossible to visualize, so formulas like the Euler characteristic (called **invariants)** are extremely useful in telling shapes apart.

Euler's original finding in 1752 was that all simple 3 dimensional shapes, called **polyhedra**, have the same Euler characteristic. A polyhedron is a 3 dimensional shape that has vertices, edges, and faces such each edge connects two faces.



Notice that, if these shapes were made of rubber, we could reshape them to look like each other. Thus, they are topologically the same type of shape. In particular, they are the same as example 2.2.

Since they are all the same, they should all have the same Euler characteristic. What is it?