Week #14 Homework: Due on Wednesday, November 28.

1. Consider an N-period binomial short -rate model with interest rate process $\{R_n\}_{n=0}^{N-1}$. Assume that $R_n(\omega) > 0$ for all $n \in \{0, \ldots, N-1\}$ and all $\omega \in \Omega$. The risk neutral probability measure is a binomial prodict measure with probability of heads 1/2.

   For a given $K > 0$ let V be an American derivative security with maturity N and intrinsic value process $\{G_n\}_{n=0}^{N}$ given by

   $$G_0 = 0 \quad \text{and} \quad G_n = (R_{n-1} - K)^+, n > 0.$$

   Either prove that it is never optimal to exercise V prior to maturity or give an example in which early exercise is optimal.

2. This exercise builds on an example from class. Consider a 3-period binomial model with interest rate process given by

   $$R_0 = .06, \quad R_1(H) = .066, \quad R_1(T) = .052$$

   $$R_2(HH) = .08, \quad R_2(HT) = R_2(TH)) = .07, \quad R_2(TT) = .05$$

   (a) Consider a security that pays $10,000R_2$ at time 3. In class, we computed $\text{For}_{0,3}$ and $\text{Fut}_{0,3}$ for this security. For this exercise, compute $\text{For}_{0,2}$, $\text{Fut}_{0,2}$, $\text{Fut}_{1,2}(H)$, $\text{Fut}_{1,2}(T)$. How do $\text{For}_{0,2}$ and $\text{Fut}_{0,2}$ compare to $\text{For}_{0,3}$ and $\text{Fut}_{0,3}$

   (b) Now consider a security that pays $10,000R_2$ at time 2. Determine $\text{For}_{0,2}$, $\text{Fut}_{0,2}$, $\text{Fut}_{1,2}(H)$, $\text{Fut}_{1,2}(T)$ for this security.

3. Consider a 3-period Ho-Lee model

   $$R_n(\omega_1 \ldots \omega_n) = a_n + b \cdot \#H(\omega_1 \ldots \omega_n),$$

   with $a_n = .05 - .005n$ and $b = .01$. The risk-neutral measure in this model is a binomial product measure with probability of heads (and probability of tails) equal to 1/2.

   Let W be a zero coupon bond with maturity 3 and face value \$100. Compute forward and futures prices for delivery of this bond at time 2 (one period before maturity). Specifically, compute

   $$\text{For}_{0,2}, \quad \text{For}_{1,2}(H), \quad \text{For}_{1,2}(T)$$

   and

   $$\text{Fut}_{0,2}, \quad \text{Fut}_{1,2}(H), \quad \text{Fut}_{1,2}(T)$$

4. Consider an 11-period Ho-Lee model

   $$R_n(\omega_1 \ldots \omega_n) = a_n + b \cdot \#H(\omega_1 \ldots \omega_n),$$

   with $a_n = .02 - .00125n$ and $b = .0025$. The risk-neutral measure in this model is a binomial product measure with probability of heads (and probability of tails) equal to 1/2.

   Compute the following

(a) The forward interest rate $F_{0,10}$.

(b) $\widetilde{E}[100B_{10,11}]$.

(c) $\widetilde{E}[100(1 - 4R_{10})]$.