Homework #3: Due on Wednesday, January 29.

Spring 2020

- 1. (omit)
- 2. (omit)
- 3. Consider a simple financial model with two times, t = 0, 1, a single stock, S, which pays no dividends, and a one period interest rate r = .08. The initial price per share of the stock is  $S_0 = $40$ . Consider a contract that requires it's owner to receive one share of stock in exchange for a payment of K at time t = 1.
  - (a) What is the value of such a contract at t = 0 if K = \$45? If K = \$35?
  - (b) Find the payment K that makes the value of the contract at t = 0 equal to zero. This value of K is called the *forward price* of the stock at time 0 for delivery at time 1, denoted  $\mathcal{F}_{0,1}$ .
- 4. (omit)
- 5. Consider a simple financial market with three times t = 0, 1, 2 and a domestic currency, say dollars, and a foreign currency, say British pounds. In this model, we can
  - Exchange any amount of dollars and pounds at t = 0 at the exchange rate  $E_{\$}^{\pounds} = 1.5$ , i.e. it costs \$1.50 to purchase one pound at time 0.
  - Borrow or invest any amount of dollars between t = 0 and t = 1 at the one-period interest rate  $r_0^{\$} = .08$  and borrow or invest any amount of dollars between t = 1 and t = 2 at the one-period interest rate  $r_1^{\$} = .12$ . An amount  $\alpha$  invested at t = i will grow to the amount  $\alpha(1 + r_i^{\$})$  at t = i + 1. Similarly for loans. (In particular, an amount  $\alpha$  invested at t = 0 and left in the bank until t = 2 will grow to  $\alpha(1.08)(1.12)$ at t = 2.)
  - Borrow or invest any amount of pounds between t = 0 and t = 1 at the one-period interest rate  $r_0^{\pounds} = .10$  and borrow or invest any amount of pounds between t = 1 and t = 2 at the one-period interest rate  $r_1^p = .15$ .

Consider a contract made between two investors A and B at t = 0 in which it is agreed that Investor A will pay Investor B \$2 at each of the times t = 1 and t = 2 and Investor B will pay Investor A  $\pounds 2$  at each of the times t = 1 and t = 2. Find the arbitrage-free price, in dollars, of Investor A's position at t = 0.