Week #11 Written Assignment: Due on *Friday*, November 8.

1. Use the rules for vector addition and scalar multiplication, and the rule for matrix multiplication to show that

   (a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \left(\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

   (b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \left(r \begin{bmatrix} u \\ v \end{bmatrix} \right) = r \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)$

2. Suppose that $\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}$ is a solution to the linear system of differential equations

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   and that $r \in \mathbb{R}$. Show that

$$r \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = \begin{bmatrix} r\phi_1(t) \\ r\phi_2(t) \end{bmatrix}$$

   is also a solution to the system.

3. Suppose that $\begin{bmatrix} A \\ B \end{bmatrix}$ is an eigenvector of of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with corresponding eigenvalue λ, i.e.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \lambda \begin{bmatrix} A \\ B \end{bmatrix}.$$

   Show that $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = e^{\lambda t} \begin{bmatrix} A \\ B \end{bmatrix}$ is a solution to the system

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$