

MA 355 Homework 9

1 Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

2 Let $D \subset \mathbb{R}$. Let $f : D \rightarrow \mathbb{R}$ be uniformly continuous on D and suppose $\{x_n\}$ is a Cauchy sequence in D . Then $\{f(x_n)\}$ is a Cauchy sequence.

3 Let $D \subset \mathbb{R}$. Let $f : D \rightarrow \mathbb{R}$ be uniformly continuous on the bounded set D . Prove that f is bounded on D .

(Hint: First show there is a sequence $s_n \in D$ such that $f(s_n) \geq n, \forall n$.)

#4 Use the definition of derivative to find the derivative of $f(x) = \sqrt{x}$ for $x > 0$.

#5 Let $f(x) = x^2 \sin\left(\frac{1}{x^2}\right)$ for $x \neq 0$ and $f(0) = 0$.

a) Show that f is differentiable in \mathbb{R} .

b) Show that f' is not bounded on the interval $[-1, 1]$.