

MA 355 Homework 8

#1 Define  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = x^2 - 3x + 5$ . Use the definition (of continuity) to prove that  $f$  is continuous at 2.

#2 Prove: Let  $D \subset \mathbb{R}$ . Let  $f : D \rightarrow \mathbb{R}$  be continuous at  $c \in D$ . Prove that there exists an  $M > 0$  and a neighborhood  $U$  of  $c$  such that  $|f(x)| \leq M$  for all  $x \in U \cap D$ .

#3 Prove: Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function and let  $k \in \mathbb{R}$ . Prove that the set  $f^{-1}(\{k\})$  is closed.

#4 Suppose  $f$  is a real function defined on  $\mathbb{R}$  which satisfies  $\lim_{h \rightarrow 0} [f(x+h) - f(x-h)] = 0$  for every  $x \in \mathbb{R}$ . Does this imply  $f$  is continuous?

#5 If  $f$  is a continuous mapping of a metric space  $X$  into a metric space  $Y$ , prove that  $f(\overline{E}) \subset \overline{f(E)}$  for every set  $E \subset X$ .

# 6 Show the equation  $3^x = x^2$  has at least one real solution.