MA 355   Homework 7

#1 Use the definition to prove $\lim_{x\to 5} x^2 - 3x + 1 = 11$.

#2 Let $D \subset \mathbb{R}$. Let f, g, h be functions from D into $\mathbb{R}$ and let c be a limit point of D. Suppose $f(x) \leq g(x) \leq h(x)$, for all $x \in D$ with $x \neq c$, and suppose that $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$. Prove that $\lim_{x\to c} g(x) = L$.

#3 Prove: Let $D \subset \mathbb{R}$. If $f : D \to \mathbb{R}$ and if c is a limit point of D, then f can have only one limit at c.

#4 Prove: Let $D \subset \mathbb{R}$. If $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a} |f(x)| = |L|$. What about the converse? (Hint: First prove $||x| - |y|| < |x - y|$)