

**MA 355      Homework 6**

#1 Prove the sequence  $s_1 = 1$ ,  $s_{n+1} = \frac{1}{4}(s_n + 5)$  where  $n \in \mathbb{N}$  is monotone and bounded. Then find the limit.

#2 Find the upper and lower limits of the sequence  $\{s_n\}$  defined by

$$s_1 = 0, \quad s_{2m} = \frac{s_{2m-1}}{2}, \quad s_{2m+1} = \frac{1}{2} + s_{2m}.$$

#3 Find an example of a sequence of real numbers satisfying each set of properties:

- a) Cauchy but not monotone
- b) Monotone but not Cauchy
- c) Bounded but not Cauchy

#4 For any two real sequences  $\{a_n\}, \{b_n\}$ , prove that

$$\limsup_{n \rightarrow \infty} (a_n + b_n) \leq \limsup_{n \rightarrow \infty} a_n + \limsup_{n \rightarrow \infty} b_n$$

Assuming the right hand side is not of the form  $\infty - \infty$ .

#5 Prove: A monotone decreasing sequence is convergent iff it is bounded.