MA 355 Homework 5 solutions

#1 Use the definition of convergence to show $lim_{n\to\infty}\frac{3n+1}{n+2}=3$.
Let $\varepsilon>0$. Choose $N=\frac{5}{\varepsilon}$ then for $n>N$, $|\frac{3n+1}{n+2}-3|<\frac{5}{n}<\frac{5}{N}<\varepsilon$.

2 Show $\lim_{n\to\infty}\frac{\sqrt{n}}{n+1}=0$.
Let $\varepsilon>0$. Choose $N=\frac{1}{\varepsilon^2}$. The for $n>N$, $|\frac{\sqrt{n}}{n+1}|<\frac{\sqrt{n}}{n}>\frac{1}{\sqrt{n}}<\frac{1}{\sqrt{N}}=\varepsilon$.

3 Determine is the following sequences diverge or converge. Find any limits that exist. Support your answers.
a) $s_n=\frac{3-2n}{1+n}=\frac{\frac{3}{n}-2}{\frac{1}{n}+1}\to -2$, since $lim\frac{1}{n}=0$.
b) $s_n=\frac{(-1)^n n}{2n-1}=\frac{(-1)^n}{2-\frac{1}{n}}$, which does not converge.
c) $s_n=\sqrt{n^2+n}-n=\frac{n}{\sqrt{n^2+n}+n}=\frac{1}{\sqrt{1+\frac{1}{n}}+1}\to\frac{1}{2}$

4 a) Give an example of a convergent sequence $\{s_n\}$ of positive numbers such that $\lim\frac{s_{n+1}}{s_n}=1$.
$s_n=1$.
b) Give an example of a divergent sequence $\{s_n\}$ of positive numbers such that $\lim\frac{s_{n+1}}{s_n}=1$.
$s_n=n$.

5 Suppose $\{s_n\}$ and $\{t_n\}$ are real sequences and $\lim_{n\to\infty}s_n=s$. Show $\lim ks_n=ks$ and $\lim(k+s_n)=k+s$ for all $k\mathbb{R}$.
a) $|(k+s_n)-(k+s)|=|s_n-s|<\varepsilon$.
b) $\lim_{n\to\infty}s_n=s$, there exists a N such that for all $n\geq N$, $|s_n-s|<\frac{\varepsilon}{|k|}$. So $|ks_n-ks|=|k||s_n-s|<|k|\frac{\varepsilon}{|k|}=\varepsilon$.

6 Prove that if $\{s_n\}$ converges then $\{|s_n|\}$ converges.
Suppose $s_n\to s$. Then $\forall\varepsilon>0\exists N$ such that $|s_n-s|<\varepsilon$ when $n>N$. But $||s_n|-|s||\leq|s_n-s|<$ $varepsilon$.

7 Suppose there exists N_0 such that $s_n\leq t_n$ for all $n>N_0$. Prove that if $\lim s_n=+\infty$, then $\lim t_n=+\infty$.
Let $M\in\mathbb{R}$, let N_1 be such that $n>N_1\implies s_n>M$. Such an N_1 exists since $\lim s_n=+\infty$. Now let $N=max\{N_0,N_1\}$. Then $n>N\implies M<s_n\leq t_n$, so $\lim t_n=+\infty$.

8 Show $\lim n^2=+\infty$.
Given $M\in\mathbb{R}$, let $N=|M|$. Then for $n>N$ we have $n^2\geq n>N\geq M$.