MA 355 Homework 4

#1 Prove: If S is a nonempty closed, bounded subset of $\mathbb{R}$, then S has a maximum and a minimum.

#2 Let $K = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$. Prove that K is compact directly from the definition.

3 Give an example of an open cover of the segment $[0, 1)$ which has no finite subcover.

4 Let E be the set of all $x \in [0, 1]$ whose decimal expansion contains only the digits 4 and 7. Is E countable? Is E dens in [0,1]? Is E compact?

5 Show that the Bolzano-Weierstrass theorem does not necessarily hold in all arbitrary ordered fields F. (Hint: Consider $F = \mathbb{Q}$)

6 Are closures and interiors of connected sets always connected? (Hint: Look at subsets of $\mathbb{R}^2$)