MA 355 Homework 2

#1 Let A, B, C be sets and let $f : A \to B$, $g : B \to C$ be functions. Prove: If f is onto B and g is onto C, then $g \circ f : A \to C$ is onto C.

2 Show the relation $\sim$ (two sets are equivalent) is an equivalence relation.

#3 Give an example of a countable collection of finite sets whose union is not finite.

4 Are the following sets finite, countable or uncountable? Explain or prove your answer in each case.
(i) $\{(x, y) \in \mathbb{N} \times \mathbb{R} : xy = 1\}$
(ii) $\left(\frac{1}{4}, \frac{3}{4}\right)$

#5 Is the set of all irrational numbers countable? Prove your answer.

#6 Let $\mathbb{N}$ be the set of natural numbers. Prove that $\mathbb{N} \times \mathbb{N}$ is countable.
Hint: Consider $f(m, n) = 2^{m-1}(2n - 1)$.