MA 355 Homework 11

#1 Suppose that $f(x) = x$ for all $x \in [0, b]$. Show that f is integrable and that $\int_0^b f(x)dx = \frac{b^2}{2}$.

2 Suppose $f(x) = c$ for $x \in [a, b]$. Show that f is integrable and that $\int_a^b f(x)dx = c(b - a)$.

3 Suppose $f \geq 0$, f is continuous on $[a, b]$ and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.

#4 If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$.

#5 Suppose that f is integrable on $[a, b]$ and that there exists $k > 0$ such that $f(x) \geq k$ for all $x \in [a, b]$. Prove that $\frac{1}{f}$ is integrable on $[a, b]$.

#6 Prove the mean value theorem for integrals: If f is continuous on $[a, b]$, then there exists $c \in (a, b)$ such that $f(c) = \frac{1}{b-a} \int_a^b f$.

#7 Suppose f is a bounded real function on $[a, b]$, and f^2 is Riemann Integrable on $[a, b]$. Does it follow that f is integrable? Does the answer change if we assume f^3 is integrable?

#8 Let f be continuous on $[a, b]$. Suppose that $\int_a^x f = \int_x^b f$ for all $x \in [a, b]$. Prove that $f(x) = 0$ for all $x \in [a, b]$.