MA 355 Homework 1

• For each subset of $\mathbb{R}$, give its maximum, minimum, supremum, and infimum, if they exist:

$$\{1,3\}, \quad \left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}, \quad (-\infty, 4)$$

p.21: #1 If r is rational ($r \neq 0$) and x is irrational, prove that $r + x$ and rx are irrational.

2 Prove that there is no rational number whose square is 12.

4 Let E be a nonempty subset of an ordered set; suppose α is a lower bound of E and β is an upper bound of E. Prove that $\alpha \leq \beta$.

5 Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. Prove that $inf A = -sup(-A)$.

• Prove: If $x, y \in \mathbb{R}$, then there exits an irrational number w such that $x < w < y$.