1. Find the partial derivatives of the following functions.

(a) x^2+y^2

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

(b) $\sin(x+y)\cos(x-y)$

$$\frac{\partial f}{\partial x} = \cos(x+y)\cos(x-y) - \sin(x+y)\sin(x-y)$$ by the product rule

$$\frac{\partial f}{\partial y} = \cos(x+y)\cos(x-y) + \sin(x+y)\sin(x-y)$$ by the product & chain rules

(c) $e^{xy^2+yz^2+zx^2}$

$$\frac{\partial h}{\partial x} = (y^2+2zx)e^{xy^2+yz^2+zx^2}$$

$$\frac{\partial h}{\partial y} = (z^2+2xy)e^{xy^2+yz^2+zx^2}$$

$$\frac{\partial h}{\partial z} = (x^2+2yz)e^{xy^2+yz^2+zx^2}$$

2. For each of the following functions $f : \mathbb{R}^2 \to \mathbb{R}$ and points (a,b) in $\mathbb{R}^2$, find the equation of the tangent plane to the graph of f at (a,b). Draw a sketch if you can.

(a) $f(x,y) = x^2 + y^2$; $(a,b) = (1,1)$.

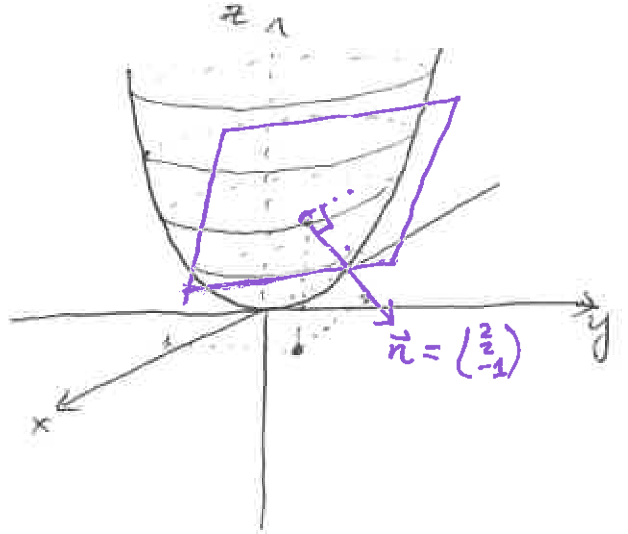
$f_x(1,1) = 2 \times 1 = 2$, $\quad f_y(1,1) = 2 \times 1 = 2$, $\quad f(1,1) = 1^2 + 1^2 = 2$

So the tangent plane is:

$z = 2 + 2(x-1) + 2(y-1)$

or equivalently:

$2x + 2y - z = 2$



(b) $f(x,y) = x\cos y - y\sin x$; $(a,b) = (0, \frac{\pi}{4})$.

$\frac{\partial f}{\partial x} = \cos y - y\cos x \Rightarrow f_x(0, \frac{\pi}{4}) = \frac{1}{\sqrt{2}} - \frac{\pi}{4}$

$\frac{\partial f}{\partial y} = -x\sin y - \sin x \Rightarrow f_y(0, \frac{\pi}{4}) = 0$

$f(0, \frac{\pi}{4}) = 0$

So the tangent plane is

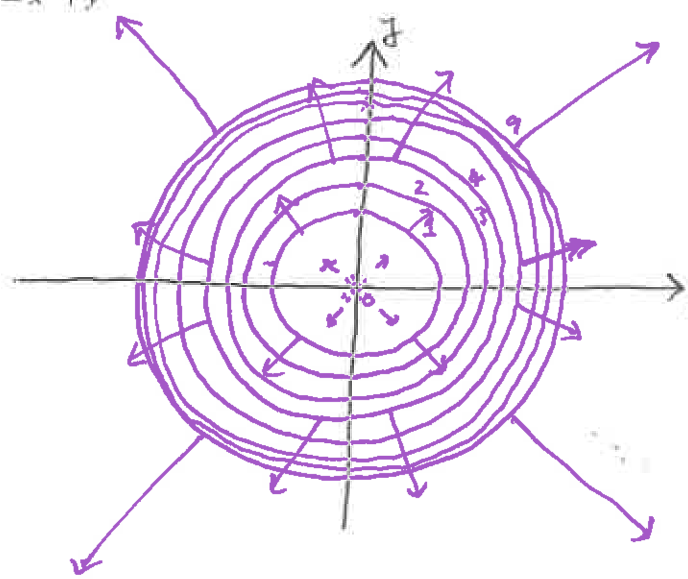
$z = 0 + \left(\frac{1}{\sqrt{2}} - \frac{\pi}{4}\right)(x - 0) + 0\left(y - \frac{\pi}{4}\right)$

i.e. $z = \left(\frac{1}{\sqrt{2}} - \frac{\pi}{4}\right)x$

(I won't try to sketch $z = x\cos y - y\sin x$...)

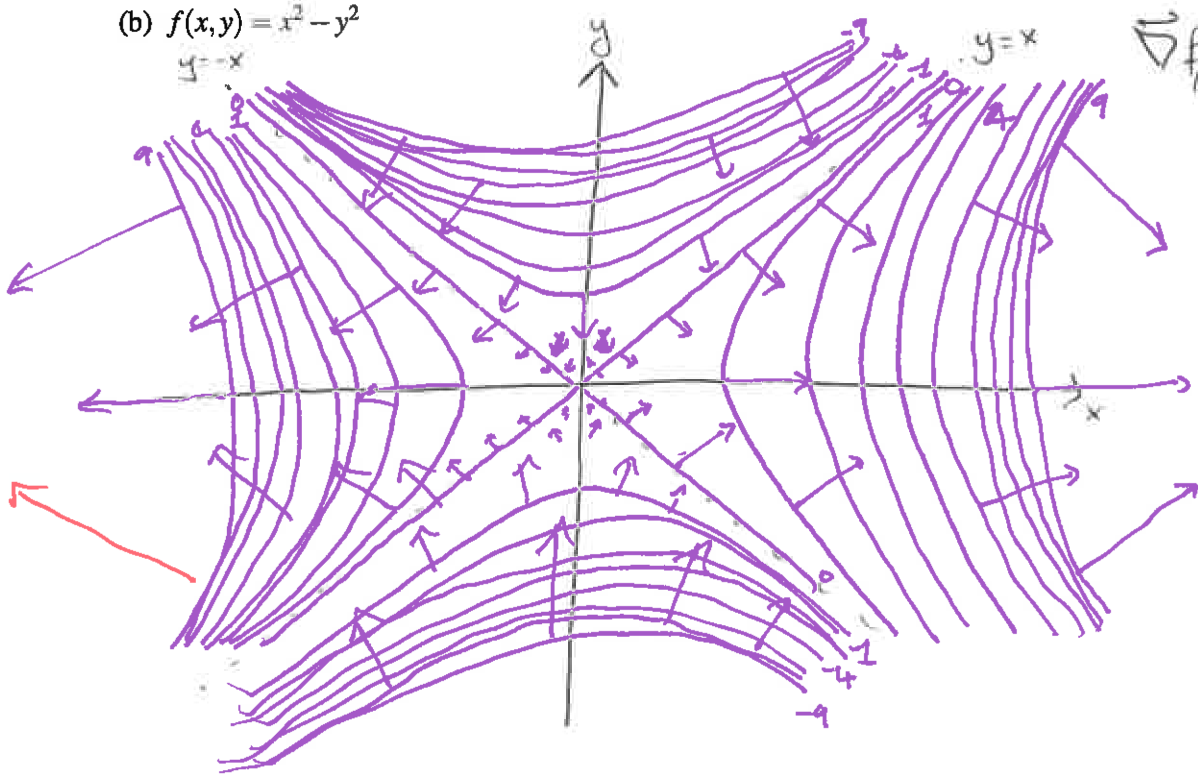
3. For each of the following functions $f : \mathbb{R}^2 \to \mathbb{R}$, sketch the level curves of its graph and indicate the direction of $\nabla f(a,b)$ at a few points (a,b) of your choosing.

(a) $f(x,y) = x^2 + y^2$



$\vec{\nabla} f(a,b) = (2a, 2b)$

(b) $f(x,y) = x^2 - y^2$



$\vec{\nabla} f_{(a,b)} = (2a, -2b)$