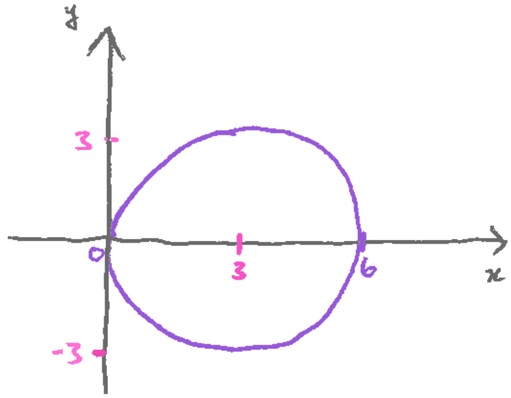
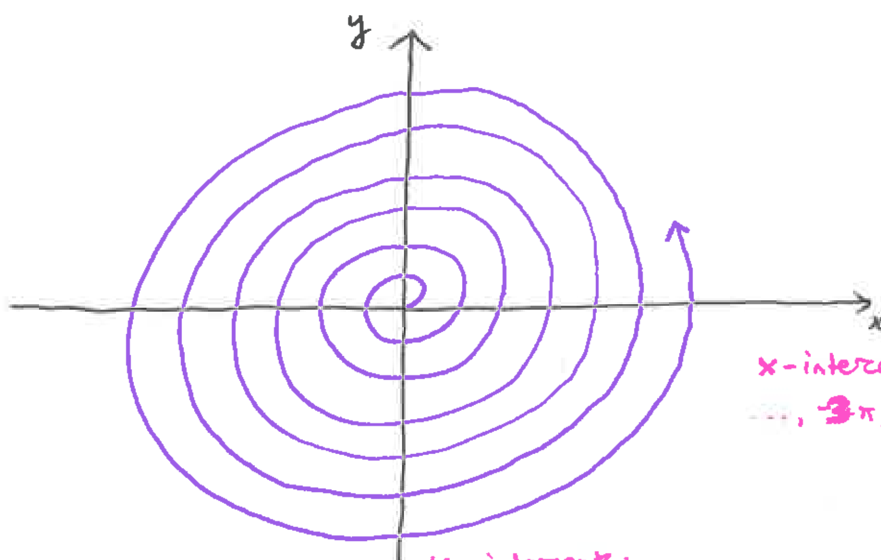
1. Find the equation of the circle $(x-3)^2 + y^2 = 9$ in polar coordinates.



$$(x-3)^2 + y^2 = 9$$
$$\iff x^2 - 6x + 9 + y^2 = 9$$
$$\iff x^2 + y^2 = 6x$$
$$\iff r^2 = 6r\cos\theta$$
$$\iff \underline{\underline{r = 6\cos\theta}}$$

[Note: $r < 0$ when $\pi < \theta < 2\pi$, so for these values of θ, the point (r, θ) lies on the ray opposite the ray at angle θ.]

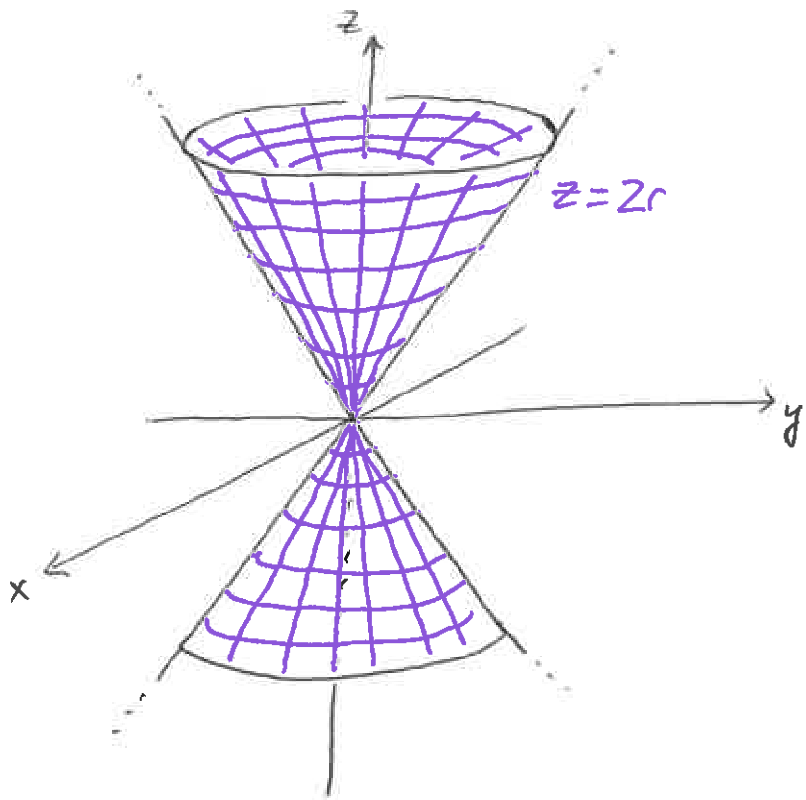
2. Sketch the curve in $\mathbb{R}^2$ described in polar coordinates by the equation $r = \theta$ (for $r \geqslant 0$).



x-intercepts:
$\ldots, -3\pi, -\pi, 0, 2\pi, 4\pi, \ldots$

y-intercepts:
$\ldots, -\frac{7\pi}{2}, -\frac{3\pi}{2}, 0, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

3. (a) Sketch the surface in $\mathbb{R}^3$ whose equation in cylindrical coordinates is $z = 2r$.



(b) Find the cartesian coordinates of the surface you just sketched.

$$z = 2r \iff z^2 = 4r^2$$
$$\iff z^2 = 4(x^2+y^2)$$
$$\iff 4x^2 + 4y^2 - z^2 = 0$$

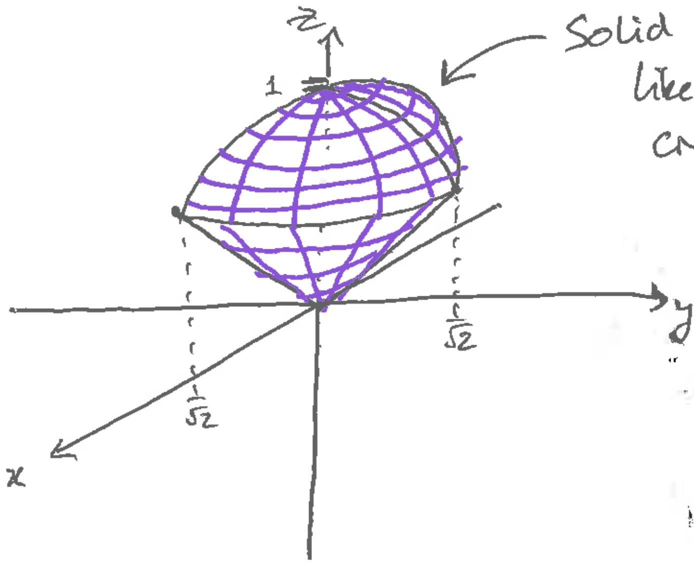
[Note: $4x^2 + 4y^2 - z^2 = \vec{x}^T A \vec{x}$

where $A = \begin{pmatrix} 4 & & \\ & 4 & \\ & & -1 \end{pmatrix}$

The surface $z=2r$ is therefore a (degenerate) hyperboloid — more on this next week]

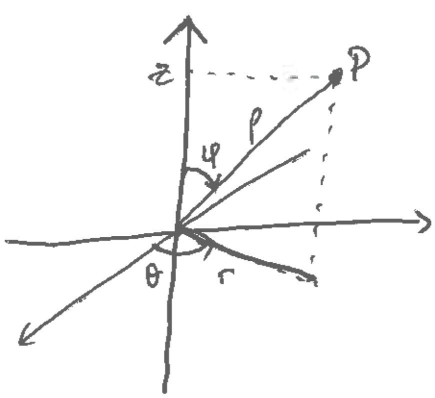
4. Sketch the solid region of $\mathbb{R}^3$ described in spherical coordinates by

$$0 \leqslant \rho \leqslant 1, \quad 0 \leqslant \varphi \leqslant \frac{\pi}{4}, \quad 0 \leqslant \theta \leqslant \pi$$



Solid region. Looks like a solid ice cream cone that has been chopped in half vertically.

5. Find a way of converting between cylindrical and spherical coordinates.



$$(r, \theta, z) \longleftrightarrow (\rho, \varphi, \theta)$$

$\theta = \theta$ (that was easy!)

$z = \rho \cos \varphi$

$r = \rho \cos(\frac{\pi}{2} - \varphi) = \rho \sin \varphi$

$\Rightarrow \tan \theta = \frac{r}{z}$ & $\rho^2 = r^2 + z^2$

Cylindrical to spherical

$$\begin{cases} \rho^2 = r^2 + z^2 \\ \tan \varphi = r/z \\ \theta = \theta \end{cases}$$

Spherical to cylindrical

$$\begin{cases} r = \rho \sin \theta \\ z = \rho \cos \theta \\ \theta = \theta \end{cases}$$