MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#8

GSI: CHRISTOPHER EUR, DATE: 10/20/2017

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Problem 1. (2 points each) If true, prove the statement. If false, give a counterexample.
(a): Every 2×2 matrix A with $\det A = 3$ is diagonalizable.
(b): Let $T : V \to V$ be a linear map. If $u, v \in V$ are eigenvectors of T, then so is $u + v$.

Problem 2. (6 points) Consider a linear map $T : \mathbb{P}_2 \to \mathbb{P}_2$ given by $p(t) \mapsto ((t+1)p(t))'$. Find a basis B of $\mathbb{P}_2$ such that the matrix of the linear transformation ${}_B[T]_B$ is diagonal.

#1. (a) False : $\begin{bmatrix} \sqrt{3} & 1 \\ 0 & \sqrt{3} \end{bmatrix}$.

(b) False : $T: \mathbb{R}^2 \xrightarrow{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}} \mathbb{R}^2$. Let $u = e_1$, $v = e_2$.

$T(u+v) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \neq \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for some λ.

#2. Let $E = \{1, t, t^2\}$ basis of $\mathbb{P}_2$.

$T(1) = (t+1)' = 1$
$T(t) = (t^2+t)' = 2t+1$
$T(t^2) = (t^3+t^2)' = 3t^2+2t$

Then $[T]_E = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

Eigenval : 1, 2, 3

eigenvect : $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_E$, $\ker \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}_E \right\}$, $\ker \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}_E \right\}$.

in $\mathbb{P}_2 \Rightarrow$ 1, $t+1$, t^2+2t+1.

$\boxed{B = \{1, t+1, t^2+2t+1\}.}$ is an eigenbasis $\Rightarrow [T]_B$ diagonal

$[T]_B = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 3 \end{bmatrix}$