MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#6

GSI: CHRISTOPHER EUR, DATE: 10/6/2017

STUDENT NAME: Tutte

Problem 1. Let V be the vector space of 2×2 matrices. Let $L := \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ be a list of vectors in V.

(a) (2 points) Extend L to a basis B of V (you need not justify B you create is a basis).

(b) (3 points) Write the coordinates of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ with respect of B you created in part (a).

Problem 2. (5 points) Let $\mathbb{P}_1 := \{a_0 + a_1 t : a_0, a_1 \in \mathbb{R}\}$ be the vector space of polynomials of degree ≤ 1. Find all values of $c \in \mathbb{R}$ for which $\{1+t, 1+ct\}$ is a basis for $\mathbb{P}_1$ (with justification)

#1. (a) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$.

(b) $\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}_B$

#2. Claim: $\boxed{c \neq 1}$

① If $c \neq 1$, $1+t$ & $1+ct$ are not scalar mult. of each other. Hence $\{1+t, 1+ct\}$ lin. indep. if $c \neq 1$.

② $\dim \mathbb{P}_1 = 2$ (e.g. has basis $\{1, t\}$).
Thus, any lin. indep. set of 2 vectors is already a basis.
($\because$ Basis extension says one can extend lin. indep set to basis by adding vector to list if necessary. But if $\dim = 2$ and the lin. indep. set has two vectors already one can't add more vectors!).

(not needed for full credit)