MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#5

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Problem 1. (6 points) Let T be a linear transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ given by $p(t) \mapsto \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}$.

(a) Show that T is *not* one-to-one.

(b) Show that T is onto (Hint: show that both $\vec{e}_1$ and $\vec{e}_2$ are in the range (image) of T).

Problem 2. (4 points) Suppose $T : V \to W$ is a linear transformation that is onto. If $\{v_1, \dots, v_n\}$ spans V, show that $\{Tv_1, \dots, Tv_n\}$ spans W.

#1. (a) Consider $p(t) := t(t-1)$. Then $T(p(t)) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Hence, $\ker T \neq \{0_{\mathbb{P}_2}\}$. Thus T not 1-1. ✓

(b) $T(-t+1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $T(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$\mathbb{R}^2 = \text{span}(T(-t+1), T(t)) \subseteq \text{Image}(T) \subseteq \mathbb{R}^2$. $\therefore$ $\text{Image}(T) = \mathbb{R}^2$ ✓

#2. Suppose $w \in W$. Then $w = Tv$ for some $v \in V$ since T onto.

Then $v = c_1 v_1 + \cdots + c_n v_n$ for some $c_1, \dots, c_n$ since $\{v_1, \dots, v_n\}$ span V.

Then $w = Tv = T(c_1 v_1 + \cdots + c_n v_n) = c_1 (Tv_1) + \cdots + c_n (Tv_n)$.

This shows that an arbitrary vector $w \in W$ is a linear comb. of $\{Tv_1, \dots, Tv_n\}$. Hence, $\{Tv_1, \dots, Tv_n\}$ spans W.