GSI: CHRISTOPHER EUR, DATE: 9/8/2017

STUDENT NAME: _____Free Speech_____

Problem 1. (6 points) Test whether the following matrix $A_{3\times4}$ has the following properties: (a) columns span $\mathbb{R}^3$, (b) columns are linearly independent. Lastly, (c) explain what these results say about existence/uniqueness of the system $A\vec{x} = \vec{b}$ for an arbitrary vector $\vec{b} \in \mathbb{R}^3$.

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & -1 & 3 \\ 0 & 1 & 4 & 0 \end{bmatrix}$$

Problem 2. (4 points) Show that a set of four vectors $\{v_1, \ldots, v_4\}$ in $\mathbb{R}^4$ need not be linear independent even if any subset of three vectors are linearly independent. In other words, give an example of $v_1, \ldots, v_4 \in \mathbb{R}^4$ such that any set of three vectors from the list is linearly independent, but $\{v_1, \ldots, v_4\}$ is not. (Hint: Can 4 vectors in 3 dimensional space ever be linearly independent?)

#1. $A \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & -23 \\ 0 & 1 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 2 & -2 & 3 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 0 & 1 & 0 \\ 0 & \boxed{1} & 4 & 0 \\ 0 & 0 & \boxed{-10} & 3 \end{bmatrix}$

(a) Yes. (pivots in each row).

(b) No. ($\exists$ free col.)

(c) $Ax = b$ is consistent ($\because A$ spans ← (a)). But not unique ($\exists$ free var.) ← (b).

#2. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

(easy to see that any three col's form lin. indep set).

But all together not lin. indep. since last vect. in span of ones before