Finding the standard matrix of a linear transformation

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Recall that given a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, the **standard matrix** of T, which we denote as M_T, is a $m \times n$ matrix:

$$M_T = \begin{bmatrix} | & \cdots & | \\ T(\vec{e}_1) & \cdots & T(\vec{e}_n) \\ | & \cdots & | \end{bmatrix}$$

Question. What if we are given $T\vec{v}_1, \ldots, T\vec{v}_n$ for $(\vec{v}_1, \ldots, \vec{v}_n)$ *not* a standard basis?

Answer. Do the following procedure: Let $A := \begin{bmatrix} | & \cdots & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & \cdots & | \end{bmatrix}$ and let $B := \begin{bmatrix} | & \cdots & | \\ T\vec{v}_1 & \cdots & T\vec{v}_n \\ | & \cdots & | \end{bmatrix}$. Row reduce the matrix $[A^t : B^t]$ (*note the transpose!!!*) to $[I_n : M]$. Then M^t is the matrix M_T.

Example. Given $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $T\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, we find that M_T is $\begin{bmatrix} 2 & -1 \\ -7/3 & 4/3 \end{bmatrix}$ as follows:

We first write:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & -1 \\ 2 & 5 & -1 & 2 \end{array}\right]$$

(again, note the transposing!). This row reduces to:

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -7/3 \\ 0 & 1 & -1 & 4/3 \end{array}\right]$$

And transposing the right matrix we get $\begin{bmatrix} 2 & -1 \\ -7/3 & 4/3 \end{bmatrix}$.