MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#6

GSI: CHRISTOPHER EUR, DATE: 10/5/2016

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Problem 1. Let H be a subset of the vector space $\mathbb{R}^3$ defined as $H := \left\{ \begin{bmatrix} 3t+2s \\ -t \\ -t+s \end{bmatrix} : t, s \in \mathbb{R} \right\}$.

(a) (2 points) Express H as a column space of some matrix, and hence conclude that H is a subspace in $\mathbb{R}^3$.

(b) (2 points) Find a basis for H.

(c) (1 point) Express H as a nullspace of some matrix (Hint: the matrix will be 1×3.)

Problem 2. Let $\mathcal{P}_2 := \{a_2x^2 + a_1x + a_0 : a_2, a_1, a_0 \in \mathbb{R}\}$ be the vector space of polynomials of degree at most 2. For the following two statements, say whether it is true or false, and explain why.

(a) (2 points) If $(q_0(x), q_1(x), q_2(x))$ is a basis of P_2, then at least one among the three must be degree 1.

(b) (3 points) Suppose we have $p_0(x), p_1(x), p_2(x) \in \mathcal{P}_2$ such that $p_j(2) = 0$ for all $j = 0, 1, 2$. Then $\text{span}\{p_0(x), p_1(x), p_2(x)\} \neq \mathcal{P}_2$.

1. (a) $\boxed{H = \text{col}\left(\begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}\right)}$ as $\begin{bmatrix} 3t+2s \\ -t \\ -t+s \end{bmatrix} = t\begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} + s\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

col. space is subspace of codomain vect. sp. ($\mathbb{R}^3$ in this case).

Hence H is a subspace of $\mathbb{R}^3$.

(b) $\boxed{\left(\underbrace{\begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}}_{= \vec{v}_1}, \underbrace{\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}_{= \vec{v}_2}\right)}$

① lin. indep (not scalar of each other)

② spans (everything in H is $t\vec{v}_1 + s\vec{v}_2$)

(c) $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix} \Rightarrow$

$2a + c = 0$

$3a - b - c = 0$

c free. $\Rightarrow$ say

$c = 2 \Rightarrow a = -1 \Rightarrow b = -5$

$\boxed{\therefore \begin{bmatrix} -1 & -5 & 2 \end{bmatrix}}$

2. (a) False: witness $(1, x + x^2, x^2)$

(b) True: any linear comb $q(x) = a_0p_0(x) + a_1p_1(x) + a_2p_2(x)$ has the property that $q(2) = a_0p_0(2) + a_1p_1(2) + a_2p_2(2) = 0 + 0 + 0 = 0$.

In other words, the span of p_0, p_1, p_2 does not contain any polynomials whose value at $x = 2$ is nonzero.

In particular, $1 \notin \text{span}(p_0, p_1, p_2)$.