MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#3

GSI: CHRISTOPHER EUR, DATE: 9/16/2016

STUDENT NAME: _____

Problem 1. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map defined by $f(x, y, z) = (-y + z, x - 3y + 2z)$, and let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map which is a $\pi/2$ radians rotation (counterclockwise) around the origin.

(a) (3 points) Write down the two matrices that correspond to f and g.

(b) (3 points) Find all (x, y, z) such that $g(f(x, y, z)) = (1, -1)$. If there is no such (x, y, z), then explain why.

Problem 2. Let $f : \mathbb{R}^m \to \mathbb{R}^n$ be a *linear* map throughout this question.

(a) (1 points) Give an example of f (i.e. write down the corresponding matrix) that is onto but not one-to-one (you should pick concrete values for m, n). You need not justify your answer.

(b) (1 points) Give an example of f that is one-to-one but not onto (again, you should pick concrete values for m, n). You need not justify your answer.

(c) (2 points) Now, suppose $m = n$, and f is one-to-one. Is it necessarily true that f is also onto? Why? [Hint: let A be the standard matrix of f. What can you say about A if f is one-to-one?]