Math 54 Fall 2016: Discussion 102/105 Quiz#1 Soln.

GSI: Christopher Eur
Office hours: Th 4-6pm, 1064 Evans
Website: `https://math.berkeley.edu/~ceur/course_pages/math54f16.html`

9/2/2016

Problem 1. (5 points) Find the general solution to the following linear system of equations.

$$x_1 - x_2 - 2x_3 = 1$$
$$x_1 + 2x_2 + 4x_3 + x_4 = 7$$
$$2x_1 + 2x_4 = 10$$

Aug. matrix: $\begin{bmatrix} 1 & -1 & -2 & 0 & 1 \\ 1 & 2 & 4 & 1 & 7 \\ 2 & 0 & 0 & 2 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 5 \\ 1 & -1 & -2 & 0 & 1 \\ 1 & 2 & 4 & 1 & 7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 5 \\ 0 & -1 & -2 & -1 & -4 \\ 0 & 2 & 4 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 5 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 0 & -2 & -6 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \sim \left[\begin{array}{cccc:c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$

$\Rightarrow$ x_3 free. $x_1 = 2$, $x_4 = 3$, $x_2 + 2x_3 = 1$

$$\Rightarrow \boxed{\begin{cases} x_1 = 2 \\ x_2 = 1 - 2x_3 \\ x_3 \text{ free} \\ x_4 = 3 \end{cases}}$$

Problem 2. Suppose M is a 3×3 coefficient matrix such that the 3×4 augmented matrix

$$\left[\begin{array}{cc} & 1 \\ M & 2 \\ & 1 \end{array}\right]$$

is consistent (i.e. has a solution although not necessarily unique).

(a) (3 points) Show that a 3×4 augmented matrix $\left[\begin{array}{cc} & c \\ M & 2c \\ & c \end{array}\right]$ is also consistent for any values of c.

(b) (1 points) Give an example of M (satisfying the above properties) such that $\left[\begin{array}{cc} & 1 \\ M & 3 \\ & 1 \end{array}\right]$ is NOT consistent.

(c) (3 points) Now, suppose that the original augmented matrix $\left[\begin{array}{cc} & 1 \\ M & 2 \\ & 1 \end{array}\right]$ has a **unique** solution. Then show that for any numbers s, t, u, the augmented system $\left[\begin{array}{cc} & s \\ M & t \\ & u \end{array}\right]$ is in fact consistent.

(a) ① swapping 2 #'s then xc = xc then swapping
② xc then xd = xd then xc
③ $c\cdot(R - dR') = (cR) - d(cR')$

This means that if $\left[\begin{array}{cc} M' & \begin{matrix}\alpha \\ \beta \\ \gamma\end{matrix}\end{array}\right]$ is the reduced form of $\left[\begin{array}{cc} M & \begin{matrix}1 \\ 2 \\ 1\end{matrix}\end{array}\right]$ then $\left[\begin{array}{cc} M' & \begin{matrix}c\alpha \\ c\beta \\ c\gamma\end{matrix}\end{array}\right]$ is the reduced form of $\left[\begin{array}{cc} M & \begin{matrix}c \\ 2c \\ c\end{matrix}\end{array}\right]$.

Hence if $\begin{pmatrix}\alpha \\ \beta \\ \gamma\end{pmatrix}$ was not pivot in first then $\begin{pmatrix}c\alpha \\ c\beta \\ c\gamma\end{pmatrix}$ isn't either.

(b) $M = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ → you get $x_1 = 1$ & $2x_1 = 3$ for $\left[\begin{array}{cc} M & \begin{matrix}1 \\ 3 \\ 1\end{matrix}\end{array}\right]$

(c) Unique soln ⇒ all col. of M pivots, but M is 3×3. Hence, reduced form of M is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Thus, $\left[\begin{array}{cc} M & \begin{matrix}s \\ t \\ u\end{matrix}\end{array}\right]$ can never have the last column as pivot.