MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#9

GSI: CHRISTOPHER EUR, DATE: 4/23/2019

STUDENT NAME: Winterfell.

Problem 1. Find a general solution to the differential equation

$$y''(x) + 4y(x) = \sin x - \cos x.$$

Problem 2. Verify by computing the Wronskian that

$$\mathbf{x}_1 = \begin{bmatrix} \cos t \\ 0 \\ 0 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} \sin t \\ \cos t \\ \cos t \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} \cos t \\ \sin t \\ \cos t \end{bmatrix}$$

are linearly independent as vector functions on $(-\infty, \infty)$. Is the Wronskian nonzero for any value of t in the interval $(-\infty, \infty)$?

#1. Let $D: V \to V$, $y \mapsto y'' + 4y$ where $V = \operatorname{span} \mathcal{B}$, $\mathcal{B} = \{\sin x, \cos x\}$.

Then $[D]_{\mathcal{B}} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ so $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Has soln. $a = \frac{1}{3}$, $b = -\frac{1}{3}$.

Since $r^2 + 4 = 0 \Rightarrow r = \pm 2i$, homog. eqn soln: $\cos 2x$, $\sin 2x$.

$\therefore \frac{1}{3}\sin x - \frac{1}{3}\cos x + C_1 \sin 2x + C_2 \cos 2x \quad \forall C_1, C_2 \in \mathbb{R}$.

#2. $W(t) = \begin{vmatrix} \cos t & \sin t & \cos t \\ 0 & \cos t & \sin t \\ 0 & \cos t & \cos t \end{vmatrix} = (\cos t)(\cos^2 t - \cos t \sin t)$

$= (\cos t)^2 (\cos t - \sin t)$

$W(0) = 1^2(1-0) = 1 \neq 0 \Rightarrow$ Wronskian is not identical to zero, hence $\vec{x}_1, \vec{x}_2, \vec{x}_3$ are lin. indep.

It is zero at $t = \frac{\pi}{2}$ for example.