MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#7

GSI: CHRISTOPHER EUR, DATE: 4/9/2019

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Problem 1. Find the SVD of the matrix $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.

Problem 2. In the above problem, find a unit vector $\vec{x} \in \mathbb{R}^2$ such that the length of $A\vec{x}$ is maximized. (If you did not do problem 1, you may suppose the SVD was $U\Sigma V^T$ where Σ is a diagonal matrix with entries $\sigma_1 > \sigma_2$, and V a matrix with columns v_1, v_2, and U a matrix with columns u_1, u_2, and phrase the answer in terms of those).

(#1) $A^TA = \begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$, charpol: $(8-\lambda)(5-\lambda) - 4$

$= \lambda^2 - 13\lambda + 36$

$= (\lambda-4)(\lambda-9) \rightsquigarrow$ sing. val: 3, 2

Eigenvec. of A^TA : $\lambda = 4$, $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\lambda = 9$, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (both have norm $\sqrt{1+4} = \sqrt{5}$)

$$V = \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$AV = U\Sigma \rightsquigarrow u_1 = \frac{1}{3}Av_1 = \frac{1}{\cancel{3}\sqrt{5}}\begin{bmatrix} \cancel{3}\,1 \\ \cancel{6}\,2 \end{bmatrix}$$

$$\quad\;\; \| $$

$$U\begin{bmatrix} 3 & \\ & 2 \end{bmatrix} \qquad u_2 = \frac{1}{2}Av_2 = \frac{1}{\cancel{2}\sqrt{5}}\begin{bmatrix} \cancel{4}\,2 \\ -\cancel{2}\,1 \end{bmatrix}$$

$$\rightsquigarrow \boxed{\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}^T.}$$

(#2) The normalized eigenvec. w/ the higher eigenval. 9 is $\boxed{\frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$