MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#4

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Problem 1. Compute the characteristic polynomial of the following matrix (you do not need to factor it).

$$\begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Problem 2. Let A be a square matrix. Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0.

(1) $\det\begin{bmatrix} -\lambda & 3 & 1 \\ 3 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{bmatrix} = -\lambda(\lambda^2-4) - 3(-3\lambda-2) + (6+\lambda)$

$= -\lambda^3 + 4\lambda + 9\lambda + 6 + 6 + \lambda$

$= -\lambda^3 + 14\lambda + 12$

(2) ① Say λ is an eigenval. of A w/ eigenvec. v.

$Av = \lambda v$.

Then $0 = A^2 v = \lambda Av = \lambda^2 v$.

Since $v \neq 0$, $\lambda^2 = 0$, thus $\lambda = 0$.

② Now, 0 is an eigenvalue of A since

$\det(A^2) = (\det A)^2 = 0 \Rightarrow \det A = 0 \Rightarrow A$ not invtbl.

$\Rightarrow \ker A \neq \{0\}$.